Rewarding Learning

ADVANCED

General Certificate of Education

2018

Mathematics

Assessment Unit F3

assessing

Module FP3: Further Pure Mathematics 3

AMF31

[AMF31]

THURSDAY 21 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$, where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are given by:

$$\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$$

$$\mathbf{b} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$$

$$\mathbf{c} = 4\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$$

Evaluate $\mathbf{a} \,.\, \mathbf{b} \times \mathbf{c}$ [4]

2 **(i)** Show that

$$\sinh^{-1} x \equiv \ln(x + \sqrt{x^2 + 1})$$ [4]

(ii) Find the exact solutions of

$$\cosh^2 x = 9 + 2 \sinh x$$

giving your answers in logarithmic form. [5]

3 **(a)** Find a vector equation of the line common to the planes

$$2x - 3y + z = 4$$

$$\text{and} \quad x - 2y + 4z = 6$$ [6]

(b) Show that the lines

$$\mathbf{r} = (-2\mathbf{i} - \mathbf{j} + 7\mathbf{k}) + \lambda(\mathbf{i} + 5\mathbf{j} + 3\mathbf{k})$$

$$\text{and} \quad \mathbf{r} = (6\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) + \mu(-5\mathbf{i} + \mathbf{j} - 2\mathbf{k})$$

have no common point. [6]

4 Differentiate

$$\tan^{-1}(2x^2+3)+\cos^{-1}\frac{x}{x+1} \qquad [7]$$

5 Let

$$I_n = \int \tan^n x \, dx \qquad n \geqslant 0$$

(i) Show that

$$I_n = \frac{1}{n-1}\tan^{n-1} x - I_{n-2} \qquad n \geqslant 2 \qquad [5]$$

(ii) Hence find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^5 x \, dx \qquad [5]$$

6 Show that

$$\int e^x \sin x \, dx = Ae^x \sin(x+B) + c$$

where A and B are constants to be determined. [8]

7 An arrow is embedded in an archery target. These can be modelled by a line and a plane. The line has equation

$$\frac{x-4.2}{3}=\frac{y-3}{1}=\frac{z-1.1}{-2}$$

The plane has equation

$$7x+y-4z=25$$

(i) Find the coordinates of the point of intersection of the line and the plane. [5]

(ii) Find the angle between the line and the plane. [6]

8 **Fig. 1** below shows sketches of the graphs of $y = \tanh x$ and $y = \operatorname{sech} x$

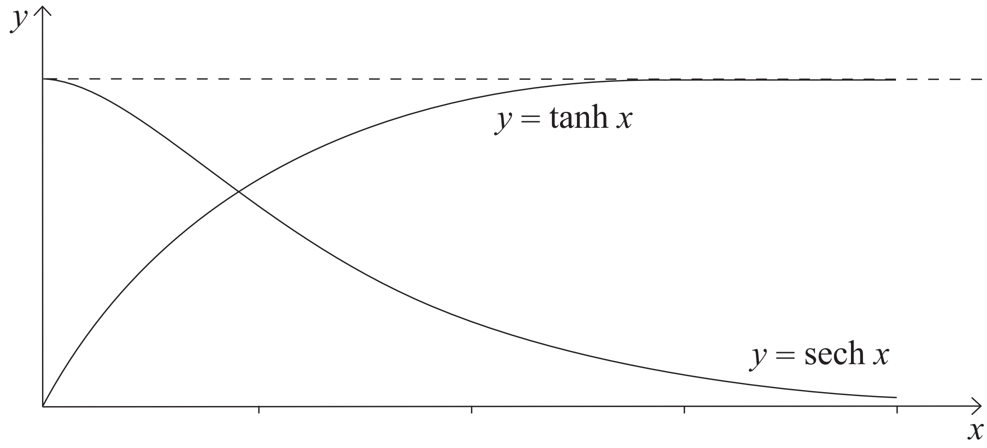

Fig. 1

(i) Prove that the graphs of $y = \tanh x$ and $y = \operatorname{sech} x$ intersect on the line $x = \ln(1 + \sqrt{2})$ [4]

(ii) The behaviour of a stock market commodity can be modelled by the exact area A between the x-axis and the curve $\mathrm{q}(x)$ defined by

$$\mathrm{q}(x) = \begin{cases} \tanh x & 0 \leqslant x \leqslant \ln(1+\sqrt{2}) \\ \operatorname{sech} x & \ln(1+\sqrt{2}) \leqslant x \leqslant \infty \end{cases}$$

Find A. [10]

THIS IS THE END OF THE QUESTION PAPER

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