Rewarding Learning

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General Certificate of Education

2018

Mathematics

Assessment Unit M3

assessing

Module M3: Mechanics 3

AMM31

[AMM31]

FRIDAY 22 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

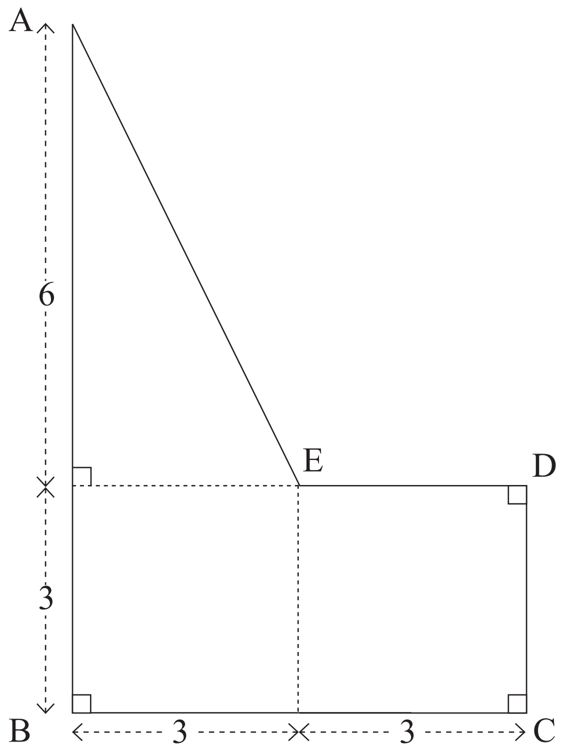
The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **Fig. 1** below shows a uniform lamina ABCDE made from uniform material of mass $1\,\text{kg}\,\text{m}^{-2}$



(All dimensions are in metres)

Fig. 1

Find the distance of the centre of mass of this lamina from the edges AB and BC. [7]

2 **Fig. 2** below shows two fixed points, P and Q, where Q is 3 m vertically below P.
A particle B of mass 2 kg is joined to P and Q by two light elastic strings, each of natural length 1 m and modulus of elasticity λ newtons.
B hangs in equilibrium $\frac{5}{3}$ m below P.

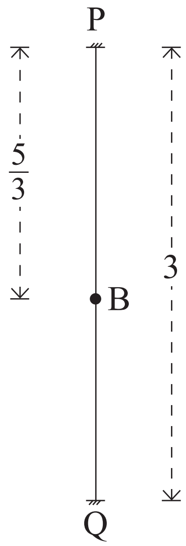


Fig. 2

(i) Show that $\lambda = 6g$. [6]

B is now pulled vertically downwards a further distance of $\frac{1}{3}$ m and released from rest.

(ii) Prove that the periodic time of the subsequent motion of B is

$$2\pi\sqrt{\frac{1}{6g}} \text{ seconds}$$ [7]

3 A ship A is sailing at a constant speed of $20\,\text{km}\,\text{h}^{-1}$ on a bearing of 090°
At noon a ship B is 35 km from A on a bearing of 150°
In order to intercept A, B sails at a constant speed of $18\,\text{km}\,\text{h}^{-1}$ on a bearing of $\theta°$

(i) Find the two possible values of θ. [8]

(ii) Find the earliest time at which B will intercept A. [5]

4 **Fig. 3** below shows three light uniform rods each of length $3a$ metres joined together in the shape of an equilateral triangle ABC.
D and E are points on AB and AC respectively, such that AD = AE = $2a$ metres.
M is the midpoint of BC.

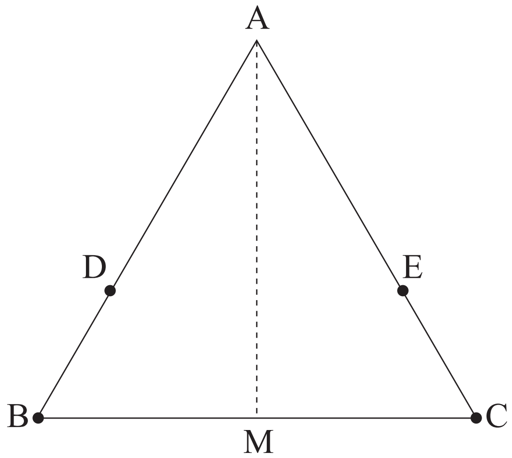


Fig. 3

Particles of mass m, $5m$, $2m$ and km are fastened at the points B, D, E and C, respectively.
The centre of mass, G, of this system of four particles lies on AM.

(i) Find k. [5]

(ii) Hence find the distance AG in terms of a. [5]

5 P and Q are fixed points with position vectors

$$\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} \text{m} \quad \text{and} \quad \begin{pmatrix} -4 \\ 2 \\ 10 \end{pmatrix} \text{m}$$

respectively.
A particle T of mass 6 kg moves along the line PQ, passing through P with a speed of $3\,\text{m}\,\text{s}^{-1}$
T is acted on by forces

$$\mathbf{F_1} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \text{N} \quad \text{and} \quad \mathbf{F_2} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \text{N}$$

The resultant force acts from P towards Q.

(i) Show that

$$\mathbf{F_2} = \begin{pmatrix} -6d-3 \\ 3d+1 \\ 6d-2 \end{pmatrix} \text{N}$$

where d is an unknown constant. [6]

T passes through Q with a speed of $12\,\text{m}\,\text{s}^{-1}$

(ii) Use the work–energy principle to find d. [5]

6 A particle P of mass m kg is fastened to one end of a light elastic string of natural length l metres and modulus of elasticity $\frac{3\sqrt{3}\,mg}{2}$ N.

The other end of the string is fastened to a fixed point Q at the top of a rough plane inclined at 60° to the horizontal.

P rests on the plane a distance $\frac{3l}{2}$ from Q down a line of greatest slope of the plane.

P is about to slide up the plane.

(i) Draw a diagram to show all the external forces acting on P. [2]

(ii) Show that the coefficient of friction between P and the plane is $\frac{\sqrt{3}}{2}$ [7]

When P is held on the plane a distance l from Q down a line of greatest slope and released from rest, it slides down the plane.

(iii) Use the work–energy principle to find the distance of P from Q when P next comes to rest. [12]

THIS IS THE END OF THE QUESTION PAPER