Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2018

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

TUESDAY 22 MAY, AFTERNOON

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

		AVAILABLE MARKS
1 **(i)** $\begin{vmatrix} 2-6 & p \\ 1 & 5-6 \end{vmatrix} = 0$	M1 W1	
$\Rightarrow 4 - p = 0$	MW1	
$\Rightarrow p = 4$	W1	
(ii) $\begin{vmatrix} 2-\lambda & 4 \\ 1 & 5-\lambda \end{vmatrix} = 0$	MW1	
$\Rightarrow (2-\lambda)(5-\lambda) - 4 = 0$	MW1	
$\Rightarrow 10 - 7\lambda + \lambda^2 - 4 = 0$		
$\Rightarrow \lambda^2 - 7\lambda + 6 = 0$		
$\Rightarrow (\lambda - 6)(\lambda - 1) = 0$		
$\Rightarrow$ other eigenvalue is 1	W1	
(iii) $\begin{pmatrix} 2 & 4 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 6 \begin{pmatrix} x \\ y \end{pmatrix}$	M1	
$\Rightarrow 2x + 4y = 6x$	MW1	
$x + 5y = 6y$		
$y = x$		
$\Rightarrow$ an eigenvector is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	W1	
$\Rightarrow$ a unit eigenvector is $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	MW1	11

2 **(i)**

$\times_{15}$	3	6	9	12	
3	9	3	12	6	
6	3	6	9	12	MW1
9	12	9	6	3	MW1
12	6	12	3	9	MW1

(ii) I – no change
p – rotation of 90° clockwise about centre — MW1
q – rotation of 180° clockwise about centre
r – rotation of 270° clockwise about centre — MW1

(iii)

	I	p	q	r	
I	I	p	q	r	MW1
p	p	q	r	I	MW1
q	q	r	I	p	MW1
r	r	I	p	q	MW1

(iv) G_1 and G_2 are isomorphic with a possible isomorphism being — MW1

$6 \Leftrightarrow I$, $9 \Leftrightarrow q$, $3 \Leftrightarrow p$, $12 \Leftrightarrow r$ — MW1

(v) G_1 has 2 elements which are self-inverse, but G_3 has 4 self-inverse elements. — MW1 — 12

AVAILABLE MARKS

3 **(i)** $(x-1)^2 + (y-7)^2 = 5$

$\Rightarrow$ Centre is (1, 7) MW1

$\Rightarrow$ Gradient of radius $= \frac{9-7}{2-1} = 2$ MW1

$\Rightarrow$ gradient of tangent $= -\frac{1}{2}$ MW1

$\Rightarrow y - 9 = -\frac{1}{2}(x-2)$ M1

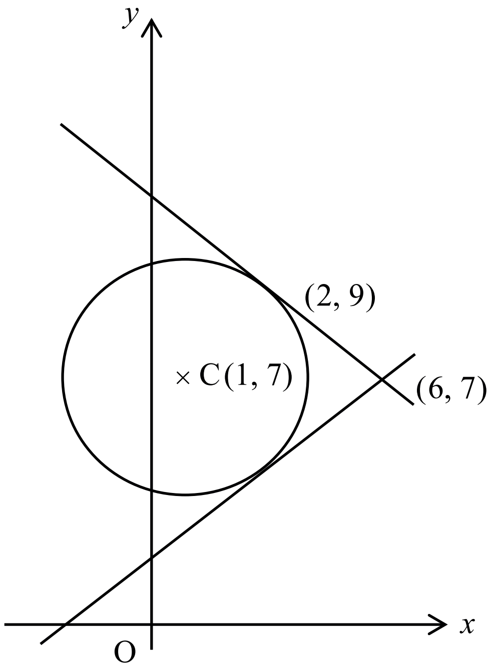
$\Rightarrow y = -\frac{1}{2}x + 10$ W1

(ii) $7 = -\frac{1}{2}(6) + 10$

$\Rightarrow 7 = 7$

Hence (6, 7) lies on the tangent. MW1

(iii)



By symmetry, the other tangent has gradient $\frac{1}{2}$ M1 W1

$\Rightarrow y - 7 = \frac{1}{2}(x-6)$ M1

$\Rightarrow y = \frac{1}{2}x + 4$ W1 10

					AVAILABLE MARKS
4	**(a)**	**(i)**	$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} -1 & -4 \\ -5 & -19 \end{pmatrix}$	M1	
			$\Rightarrow a + b = -1$ and $c + d = -5$ $3a + 2b = -4$ $\quad 3c + 2d = -19$	MW1	
			$\Rightarrow 3a + 3b = -3$ $\quad \Rightarrow 3c + 3d = -15$ $3a + 2b = -4$ $\quad 3c + 2d = -19$	M1	
			$\Rightarrow b = 1$ $\quad \Rightarrow d = 4$	W1	
			$a = -2$ $\quad c = -9$	W1	
			Hence matrix is $\begin{pmatrix} -2 & 1 \\ -9 & 4 \end{pmatrix}$		
		(ii)	$\begin{pmatrix} -2 & 1 \\ -9 & 4 \end{pmatrix}\begin{pmatrix} x \\ mx \end{pmatrix} = \begin{pmatrix} t \\ mt \end{pmatrix}$	M1	
			$\Rightarrow -2x + mx = t$ $-9x + 4mx = mt$	MW1	
			Divide to give		
			$\dfrac{-2 + m}{-9 + 4m} = \dfrac{1}{m}$	M1	
			$\Rightarrow -2m + m^2 = -9 + 4m$	MW1	
			$\Rightarrow m^2 - 6m + 9 = 0$ $\Rightarrow (m - 3)^2 = 0$ $\Rightarrow m = 3$		
			Hence the equation of the invariant line is $y = 3x$	W1	
	(b)	**(i)**	Rotation of 240° anticlockwise about the origin	MW1 MW1	
		(ii)	$\mathbf{A}^n$ represents a rotation through a multiple of 360°	MW1	
			Hence $n = 3$	MW1	14

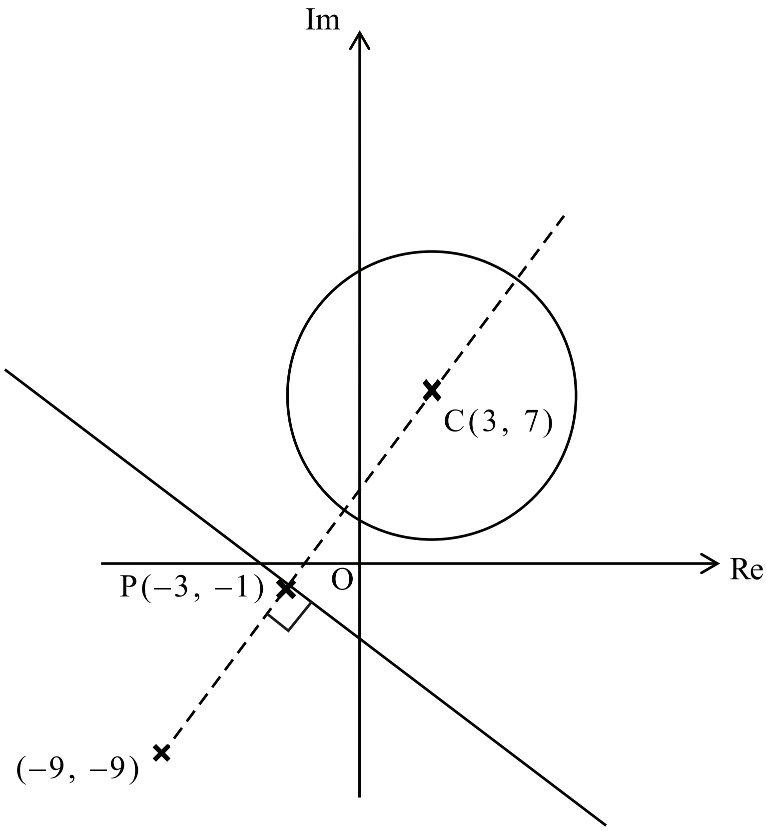
5 **(a)** $\dfrac{3+4i}{1-7i} \times \dfrac{1+7i}{1+7i}$ M1

$= \dfrac{3+4i+21i-28}{1+49}$ MW1

$= \dfrac{-25+25i}{50}$ W1

$= -0.5 + 0.5i$

(b) **(i)** Circle, centre (3, 7) and radius 6 MW3



(ii) Perpendicular bisector of the line joining (–9, –9) and (3, 7) MW3

(iii) Minimum value of $|z-w|$ = PC – 6 P = (–3, –1) M1

PC = $\sqrt{6^2+8^2} = 10$ MW1

Hence distance = 10 – 6 = 4 W1 | 12

AVAILABLE MARKS

6 (a) (i) $\mathbf{AB} = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 7 & -2 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ -1 & 3 \\ 0 & 4 \end{pmatrix}$ M1

$= \begin{pmatrix} 1 & 25 \\ -5 & 13 \end{pmatrix}$ W1

(ii) $\mathbf{BA} = \begin{pmatrix} 2 & 0 \\ -1 & 3 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 2 & 3 & 4 \\ 1 & 7 & -2 \end{pmatrix}$

$= \begin{pmatrix} 4 & 6 & 8 \\ 1 & 18 & -10 \\ 4 & 28 & -8 \end{pmatrix}$ MW1

Since $\mathbf{AB} \neq \mathbf{BA}$, then the commutative law does not hold. MW1

Alternative Solution

A is a 2 × 3 matrix and **B** is a 3 × 2 matrix.

Hence **AB** will have order 2 × 2 and **BA** will have order 3 × 3 MW1

Therefore, $\mathbf{AB} \neq \mathbf{BA}$ and the commutative law does not hold. MW1

(b) (i) $\begin{vmatrix} \lambda & 1 & 0 \\ 3 & -2 & \lambda - 3 \\ 10\lambda & 3 & -2 \end{vmatrix} \neq 0$ M2

$\Rightarrow \lambda\{4 - 3(\lambda - 3)\} - 1\{-6 - 10\lambda(\lambda - 3)\} \neq 0$ MW1

$\Rightarrow 4\lambda - 3\lambda(\lambda - 3) + 6 + 10\lambda(\lambda - 3) \neq 0$

$\Rightarrow 4\lambda - 3\lambda^2 + 9\lambda + 6 + 10\lambda^2 - 30\lambda \neq 0$

$\Rightarrow 7\lambda^2 - 17\lambda + 6 \neq 0$

$\Rightarrow (7\lambda - 3)(\lambda - 2) \neq 0$ W1

$\Rightarrow \lambda \neq \frac{3}{7}, \quad \lambda \neq 2$ W1

(ii) $2x + y = u$ ① MW1

$3x - 2y - z = w$ ②

$20x + 3y - 2z = 15$ ③

② × 2 $\Rightarrow 6x - 4y - 2z = 2w$

③ $\Rightarrow 20x + 3y - 2z = 15$

③ – ② × 2 $\Rightarrow 14x + 7y = 15 - 2w$

① × 7 $\Rightarrow 14x + 7y = 7u$ MW1

Therefore, for no real solutions $7u \neq 15 - 2w$ MW1

(iii) $2x + y = 1$ ①

$3x - 2y - z = 4$ ②

$20x + 3y - 2z = 15$ ③

② × 2 $\Rightarrow 6x - 4y - 2z = 8$ MW1

③ – ② × 2 $\Rightarrow 14x + 7y = 7$ which is consistent with ①

$\Rightarrow y = 1 - 2x$ M1

Hence ② gives $3x - 2(1 - 2x) - z = 4$

$\Rightarrow 7x - z = 6$

$\Rightarrow z = 7x - 6$ W1

Therefore, the general solution is $(t, 1 - 2t, 7t - 6)$ MW1 16

	AVAILABLE MARKS
Total	**75**