New Specification

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education

Mathematics

Assessment Unit AS 1

assessing

Pure Mathematics

[SMT11]

Assessment

MARK SCHEME

(Including Supplementary Mark Scheme to Support Teachers)

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

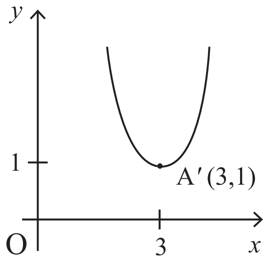
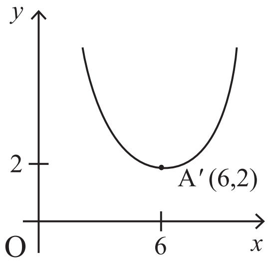
It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

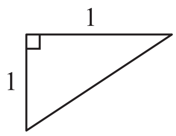
Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

			AVAILABLE MARKS
1	**(i)**  **(ii)** 	M1 W1 M1 W1	4
2	**(a)** **(i)** $2\mathbf{a} - \mathbf{b} = 2(4\mathbf{i} + 3\mathbf{j}) - (6\mathbf{i} - \mathbf{j})$	M1	
	$= 2\mathbf{i} + 7\mathbf{j}$	W1	
	(ii) $x^2 + 2^2 = 6^2$	M1 W1	
	$x^2 = 32$	MW1	
	$x = \pm\ 4\sqrt{2}$	MW1	
	(b) $f(k) = k^3 + k^2 - k + k$	M1 W1	
	$\Rightarrow \quad k^3 + k^2 = 0$	MW1	
	$k^2(k + 1) = 0$	M1	
	$k = 0 \quad k = -1$	W2	12
3	**(a)** $y = 3x + 2$	M1 W1	
	$\Rightarrow \quad x^2 + 2(3x + 2) = 20$	M1	
	$x^2 + 6x - 16 = 0$	W1	
	$(x + 8)(x - 2) = 0$	M1	
	$x = -8 \qquad x = 2$	W1	
	$y = -22 \qquad y = 8$	W1	
	(b) $\log 3^{2x-1} = \log 7^x$	M1	
	$(2x - 1)\log 3 = x \log 7$	MW1	
	$2x \log 3 - x \log 7 = \log 3$	M1 W1	
	$x(2 \log 3 - \log 7) = \log 3$		
	$x = \dfrac{\log 3}{2\log 3 - \log 7}$	M1	
	$x = 4.37$ (3 s.f.)	W1	13

				AVAILABLE MARKS
4	**(i)**	$p = 2$	MW1	
	(ii)	$-2 = \dfrac{q}{-1-2}$	M1 W1	
		$\Rightarrow q = 6$	W1	
	(iii)	$y = \dfrac{6}{-2}$	M1	
		$y = -3$		
		$(0, -3)$	W1	6
5	**(i)**	$x^2 - 2 = x$	M1	
		$x^2 - x - 2 = 0$	W1	
		$(x - 2)(x + 1) = 0$		
		$x = 2 \quad x = -1$		
		$y = 2 \quad y = -1$		
		A$(-1, -1)$ B $(2, 2)$	MW2	
	(ii)	$\int_{-1}^{0} \left(x^2 - 2\right) \mathrm{d}x = \left[\frac{x^3}{3} - 2x\right]_{-1}^{0}$	M1 W1	
		$= [0] - \left[\frac{5}{3}\right] = -\frac{5}{3}$	M1 W1	
		 Area $= \frac{1}{2}$	MW1	
		Shaded $= \frac{5}{3} - \frac{1}{2} = \frac{7}{6}$ units^2	MW1	10

			AVAILABLE MARKS
6	**(a)** $\tan 2x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	MW1	
	$\tan y = \frac{\sqrt{3}}{3} \quad 0 \leqslant y \leqslant 720°$	M1	
	$y = 30°, 210°, 390°, 570°$	W2	
	$x = 15°, 105°, 195°, 285°$	W2	
	(b) **(i)** $(4 - x)^2 = x^2 + (x + 1)^2 - 2x(x + 1)\cos 60°$	M1 W1	
	$16 - 8x + x^2 = x^2 + x^2 + 2x + 1 - x^2 - x$		
	$16 - 8x = x + 1$		
	$9x = 15$		
	$x = \frac{5}{3}$	MW1	
	(ii) Area $= \frac{1}{2}\left(\frac{5}{3}\right)\left(\frac{8}{3}\right)\sin 60°$	M1 W1	
	$= \frac{10\sqrt{3}}{9}$ units^2		
	$= 1.92$ units^2 (3 s.f.)	MW1	12
7	**(a)** $\frac{1}{5} - \frac{6}{x^3} + \frac{x^{-\frac{3}{2}}}{8}$	MW3	
	(b) **(i)** $P = 2x + 2r + \pi r$	M1 W1	
	$2x + 2r + \pi r = 32$	M1	
	$x = \frac{32 - 2r - \pi r}{2}$	W1	
	(ii) $A = xr$	MW1	
	$A = \left[\frac{32 - 2r - \pi r}{2}\right]r$	M1	
	$A = 16r - r^2 - \frac{1}{2}\pi r^2$	W1	
	(iii) $\frac{dA}{dr} = 16 - 2r - \pi r$	M1 W1	
	$16 - 2r - \pi r = 0$	M1	
	$r = \frac{16}{2 + \pi}$	W1	
	$\frac{d^2A}{dr^2} = -2 - \pi < 0 \Rightarrow$ Max	M1 W1	16

			AVAILABLE MARKS
8	$x^2 - 2x + y^2 - 6y - 3 = 0$	M1	
	$(x-1)^2 + (y-3)^2 - 1 - 9 - 3 = 0$		
	$(x-1)^2 + (y-3)^2 = 13$	W2	
	centre (1, 3)	W1	
	point (4, 5)		
	grad $= \frac{5-3}{4-1} = \frac{2}{3}$	M1 W1	
	$\text{grad}_T = -\frac{3}{2}$	MW1	
	$y = mx + c$	M1	
	$5 = 4\left(-\frac{3}{2}\right) + c$		
	$c = 11$		
	$\Rightarrow y = -\frac{3}{2}x + 11$		
	$\Rightarrow 2y = -3x + 22$		
	$\Rightarrow 3x + 2y - 22 = 0$	W1	9
9	$(1+ax)^n \equiv 1 + nax + \frac{n(n-1)(ax)^2}{2} + \cdots$	M1	
	$\equiv 1 + nax + \frac{n(n-1)a^2x^2}{2} + \cdots$	W2	
	$(1-x)(1+ax)^n \equiv 1 + nax + \frac{n(n-1)a^2x^2}{2} - x - nax^2 + \ldots$	M1	
	$\Rightarrow 1 + [na-1]x + \left[\frac{n(n-1)a^2}{2} - na\right]x^2 \equiv 1 + 23x + 228x^2$	W2	
	$na - 1 = 23$ $\quad\quad \frac{n(n-1)a^2}{2} - na = 228$	MW1	
	$a = \frac{24}{n}$ $\quad\quad \frac{n(n-1)576}{2n^2} - 24 = 228$	M1	
	$288(n-1) = 252n$		
	$36n = 288$		
	$n = 8$	W1	
	$a = 3$	W1	10
10	$8 - 6e^x + e^{2x} < 1$	M1 W1	
	$e^{2x} - 6e^x + 7 < 0$	MW1	
	CV $y^2 - 6y + 7 = 0$ $[y = e^x]$	M1	
	$(y-3)^2 - 9 + 7 = 0$		
	$(y-3) = \pm\sqrt{2}$	M1	
	$y = 3 \pm \sqrt{2}$		
	$e^x = 3 \pm \sqrt{2}$		
	$x = \ln(3 \pm \sqrt{2})$	W2	
	$\ln(3-\sqrt{2}) < x < \ln(3+\sqrt{2})$	MW1	8
		Total	**100**

Breakdown of Marks

1 **(i)** M1 Vertical shift of ± 1
W1 Correct

Notes
[A] If offer 2 solutions as vertical shift of +1 and vertical shift of –1 then award M1 only
[B] If offer 2 solutions as vertical shift of –1 and horizontal shift of –1 then award M0 W0

(ii) M1 Horizontal stretch of $\frac{1}{2}$ or 2
W1 Correct

Notes
[A] If offer 2 solutions as horizontal stretch of $\frac{1}{2}$ and horizontal stretch of 2 then M1 only
[B] If offer 2 solutions as horizontal stretch of 2 and vertical stretch of 2 then award M0W0

2 **(a)** **(i)** M1 Trying to substitute vectors **a** and **b**
W1 Correct answer

(ii) M1 Trying to use Pythagoras with vector **c**
W1 Correct
MW1 $x^2 = 32$
W1 Simplified correct answers – must have both values

(b) M1 Trying to substitute $\pm k$
W1 Correct substitution
MW1 Sets their cubic = 0 (conditional on using $f(\pm k)$
M1 Trying to factorise their cubic
W2 Each solution (cao)

Notes
[A] If use $f(-k)$ and continue their method correctly, then
award M1 W0 MW1 M1 W0 W0 i.e. 3/6

3 **(a)** M1 Trying to find x or y from linear equation
W1 Correct
M1 Trying to substitute their x, y expression into quadratic equation
W1 Correct quadratic = 0
M1 Trying to solve their quadratic equation
W1 Correct x values (cao)
W1 Correct y values (allow ft provided there are two solutions)

Notes
[A] If an early error means that the equation obtained is not quadratic, then
none of the final 4 marks may be awarded.

(b) M1 Taking logs (can use $\log_{10}$ or ln)
MW1 Correct use of 3rd rule of indices
M1 Trying to multiply out
W1 Correct
M1 Trying to factorise (provided equation has 3 terms, 2 of which involve x)
W1 Correct (cao)

4 **(i)** MW1 $p = 2$

(ii) M1 Trying to substitute for x/y
W1 Correct
W1 $q = 6$

Notes
[A] If x, y are switched i.e. $-1 = \dfrac{q}{-2 - 2}$ then award M1 only
[B] If incorrect answer from (i) is used, then can ft as M1 W1 W0 i.e. max 2/3

(iii) M1 Setting $x = 0$ into their equation from (ii)
W1 (0, –3) (cao)

5 **(i)** M1 Setting quadratic = linear
W1 Correct quadratic = 0
MW2 One mark each for A and B (cao)

(ii) M1 Trying to integrate
W1 Correctly integrated
M1 Substituting their limits (allow ft from (i))
W1 Area as $\dfrac{5}{3}$
MW1 Correct area of their triangle (allow ft from (i))
MW1 Correct answer only

6 **(a)** MW1 Correct value of tan $2x$
M1 Trying to solve their tan equation
W1 2 correct values of y
W1 2 more correct values of y
W1 2 correct values of x
W1 2 more correct values of x

Notes
[A] Uses half range only – can award MW1 M1 W1 W0 W1 W0 i.e. max of 4/6
[B] Finds tan $2x = \sqrt{3}$ – can award MW0 M1 W1(ft) W1(ft) W0 W0 i.e. max of 3/6

(b) **(i)** M1 Trying cosine rule
W1 Correct
MW1 Clear expansion and simplification to show how correct answer is obtained

Notes
[A] Correct formula for cosine rule but incorrect substitution – award M1 W0 MW0

(ii) M1 Trying to use correct area formula
W1 Correct substitution
MW1 Correct answer only

7 **(a)** MW3 One mark for each correct term

(b) **(i)** M1 Trying perimeter (3 terms each with only 1 dimension (i.e.) x term, r term and πr term)
W1 Correct
M1 Setting their perimeter equal to 32 (subject to 1st M1 being awarded)
W1 Correct x in terms of r

(ii) MW1 Using $A = xr$ (allow ft from (i))
M1 Trying to substitute for their x, r
W1 Correct answer only

(iii) M1 Trying to differentiate
W1 Each term correct
M1 Setting their derivative equal to 0
W1 Correct r
M1 Trying 2nd derivative (allow ft from their 1st derivative)
W1 Correct 2nd derivative and correct conclusion stated

8 M1 Trying to complete the square (or use of $2g = \ldots, 2f = \ldots$)
W2 One mark for each of $(x - 1)^2$ and $(y - 3)^2$ (or $2g = -2$, $2f = -6$)
W1 Correct centre (1, 3)
M1 Trying to find gradient of radius (ft their centre)
W1 Correct gradient only
MW1 Perpendicular gradient (allow ft of their radius gradient)
M1 Trying to find equation of line (ft their perpendicular gradient)
W1 Correct equation in required form

Notes
[A] If correct centre is stated without any working - 1st 4 marks can be awarded

9 M1 Trying to expand $(1 + ax)^n$
W1 First 2 terms correct
W1 Third term correct
M1 Trying to multiply by $(1 - x)$
W1 First 2 terms correct (allow ft for a constant term and an x term)
W1 Third term correct (allow ft for an x^2 term)
MW1 Equating their x term to 23 and their x^2 term to 228
M1 Trying to solve their equations (conditional on previous MW1 mark)
W1 Correct n
W1 Correct a

10 M1 Trying to expand brackets
W1 Correct
MW1 Rearranging to give disguised quadratic < 0 (allow ft)
M1 Trying to change to simple quadratic (allow ft provided a quadratic is obtained)
M1 Trying to solve their quadratic
W2 One mark for each of correct critical values (ln form or decimal)
MW1 Correct range (ln form or decimal)

Notes
[A] Error in expansion – M1 W0 and potentially MW1(ft) M1(ft) M1(ft) i.e. max 4/8