

ADVANCED General Certificate of Education 2022 Reserve Series

### **Mathematics**

Assessment Unit A2 1 assessing Pure Mathematics

### 

**Centre Number** 

**Candidate Number** 

\*AMT11\*

### [AMT11] MONDAY 27 JUNE, MORNING

#### TIME

2 hours 30 minutes.

#### **INSTRUCTIONS TO CANDIDATES**

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.

You must answer all ten questions in the spaces provided.

**Do not write outside the boxed area on each page or on blank pages or tracing paper.** Complete in black ink only. **Do not write with a gel pen**.

Questions which require drawing or sketching should be completed using an HB pencil. Show clearly the full development of your answers. **Answers without working may not gain full credit.** 

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

#### **INFORMATION FOR CANDIDATES**

The total mark for this paper is 150

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the Mathematical Formulae and Tables booklet is provided.

Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_e z$ 

### 

#### \*36AMT1101\*

1 (a) Sta	te whether the following sequences converge, diverge or oscillate.
(i)	$\frac{2n^2}{n+3}$ [1]
(ii)	$\cos\left(\frac{n\pi}{3}\right)$ [1]
(iii)	$(1) \frac{5n}{3n-1}$
13786	



\*36AMT1102\*

y Learning CC Reserved 20 J Loaming CC. 20 7 Learning Rowards 20 7 Lawriting CC. Reserved 2 Learning Research 200 G 200 7 Leventing CC. Roverda 2 Learning Roverda Reserved y Leastwing Reserved Reserved Reserved 2 Learning Research Roserte CC. G Reserved Rowardin Roards Partie 

C

*36AMT1103*
-------------

(b)	Find $\sum_{r=1}^{\infty} 3\left(\frac{2}{5}\right)^r$	[5]
		•••••
		•••••
		•••••
		···· ·
		•••••
		•••••
		•••••
	······	•••••
		•••••
	······	•••••
		•••••
		•••••
		•••••
		•••••
	······	
	[Tur	n over

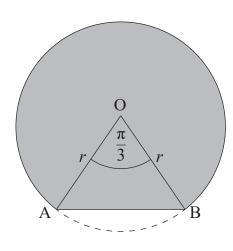
(c) The first three terms of an arithmetic progression are			
	x, $3x + 1$ , $3x^2$ where $x > 0$		
(i)	Find the value of <i>x</i> . [4]		
13786			



\*36AMT1104\*

Find the sum of the first 10 terms of this arithmetic progression. [4
•••••••••••••••••••••••••••••••••••••••
[Turn o

- DD y Learning CC. 2 Loaming CC. 20 Rowards 2D C. 20 Learning Ð Reserved 20 Rowerth 20 J Learning CC. 2D Reserts DD 7 Learning CC. a
- 2 The shaded region R, as shown in Fig. 1 below, is the major segment of a circle of radius *r* cm with angle AOB =  $\frac{\pi}{3}$  radians.







\*36AMT1106\*

The perimeter of R is  $(10\pi + 6)$  cm.

(ii) Find the value of *r*.

[2]

[5]

(iii) Find the area of R.

••••••
[Turn over



\*36AMT1107\*

$\frac{4x^2 - 25}{3x^2 + 14x + 8} \div \frac{6x - 15}{x + 4}$

y Learning CC Reserved 20 J Loaming C. 20 7 Learning Rowards 20 7 Lawriting Reverde 20 2 Learning CC. 200 C. CC. 200 7 Learning Reserved 200 7 Leventing CC. 200 7 Leventry CC. Research 20 Learning C. Roserte CC. C. Reserved C. C. 

G

\*36AMT1108\*

$\frac{x+3}{\sqrt{x+2}}$	
up to and including the term in $x^2$	[6

## 

\*36AMT1109\*

y Learning Research Pressure de Resards C. 20 2 Learning CC. 20 J Learning Rewards Rowerda y Learning CC Research 20 J Learning CC. 200 y Learning Revertin 200 C. Deaming CC. 200 7 Learning CC. CC. 200 CC. Reserved 200 7 Learning CC. 200 J Learning CC. C.

4 (a) The graph of the function y = f(x) is shown in Fig. 2 below.

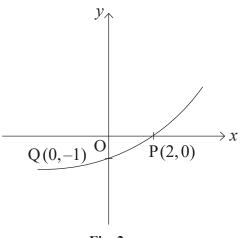
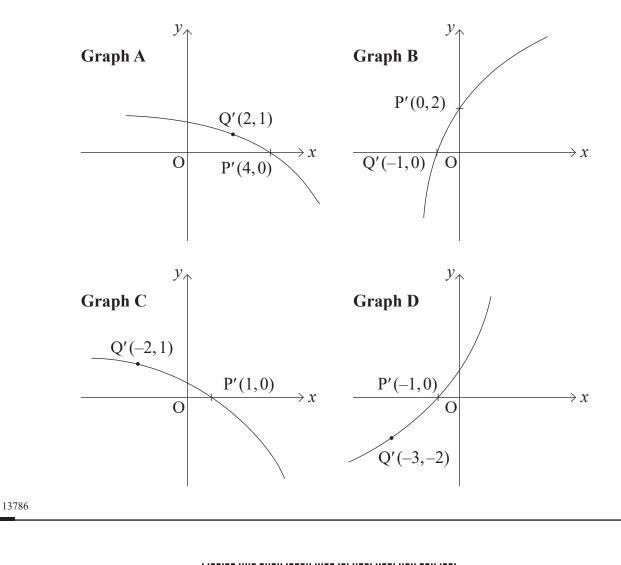


Fig. 2

The curve cuts the axes at P(2, 0) and Q(0, -1).

**Fig. 3** below shows five different transformations of y = f(x).





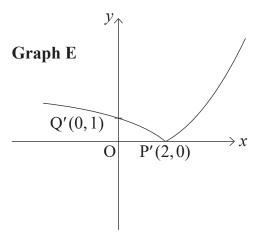




		Fig. 3	
	Complete the follo	owing statements:	
	(i) $y = 2f(x+3)$	is represented by Graph	[1]
	(ii) $y = -f(x-2)$	is represented by Graph	[1]
	(iii) $y =  \mathbf{f}(x) $	is represented by Graph	[1]
	(iv) $y = f^{-1}(x)$	is represented by Graph	[1]
13786			[Turn over

## 

#### \*36AMT1111\*

<b>(b)</b> Tl	ne functions g and h are defined by:
	$g(x) = 2 + \cos x$ $x \in \mathbb{R}$ $0 \le x \le \pi$
	$h(x) = \frac{1}{1+x} \qquad x \in \mathbb{R}  x \neq -1$
(i)	State the range of $g(x)$ . [2]
(ii	) Find the inverse function $h^{-1}(x)$ , stating its domain. [5]
(II	
3786	



\*36AMT1112\*

Find the composite function $hg(x)$ , stating its domain.	[4
	[Turn o
	-

\*36AMT1113\*

y Learning Research 22 Learning CC. D Romanda Ð a D Ca. 20 Learning Rewards C. Rasarda 200 7 Learning Rosertin 2 Lawring a

[3]

5 The points of intersection	of the curves
------------------------------	---------------

 $y = \csc^2 3x$ 

and

 $y = x^2 + 1$ 

can be found by solving the equation

13786

$$\csc^2 3x - x^2 - 1 = 0$$

(i) Show that this equation has a root between x = 0.3 and x = 0.5



\*36AMT1114\*

By taking $x = 0.3$ as a first approximation and using the Newton-Raphson method once, find a better approximation to $\alpha$ . [6]
[Turn o

# 

\*36AMT1115\*

	$\frac{\cos 2\theta - \cos \theta + 1}{\sin 2\theta - \sin \theta} \equiv \cot \theta$	[6]
••••••		
•••••		

y Learning Research

C.

## 

L

\*36AMT1116\*

(b) Solve the equation

$$\tan(\theta - 45^\circ) = 6 \tan \theta$$
 where  $0^\circ \le \theta \le 360^\circ$  [7]

..... ..... ..... ..... [Turn over

### 

\*36AMT1117\*

7 (a)	A curve is defined by the parametric equations	
	$x = \sin t + \cos t$ $y = 4 - 3 \sin 2t$	
	Find the Cartesian equation of this curve. [7	7]
		· ·
13786		



\*36AMT1118\*

C

[Turn over

y Learning Research 20 7 Learning C. 20 7 Learning CC. J.Learning Descention 200 Rosserede Posserede Posserede Rowerth Zeamay 7 Learning Research 200 7 Leventing C. C. 20 Learning CC. Romanda 20 7 Learning Resards CC. Research CC. G

(b) A curve is given by the equation

 $3x + 5xy^2 - 16 = 2(x + y)^2$ 

••••	 					•••••		 		•••••					
••••	 					•••••		 		•••••					
••••	 					• • • • • • • •		 		•••••					
••••	 					••••••		 		•••••					
••••	 					• • • • • • • •		 		•••••					
••••	 							 							
••••	 							 							
••••	 							 							
••••	 					•••••		 		•••••					
••••	 		•••••	• • • • • • • •	• • • • • • • •	• • • • • • • •	• • • • • • • • •	 	•••••	••••	• • • • • • • •	• • • • • • • • • •	• • • • • • • • •	•••••	• • • • •
••••	 					• • • • • • • •	• • • • • • • • •	 		•••••	•••••				
••••	 			• • • • • • • •	• • • • • • • •	•••••		 	•••••	••••	•••••			•••••	
••••	 •••••	•••••	•••••	• • • • • • • •	• • • • • • • •	• • • • • • • •	• • • • • • • • •	 	•••••	••••	•••••	• • • • • • • • • •	• • • • • • • • •	•••••	• • • • •
••••	 					•••••		 		•••••					
••••	 			•••••	•••••	•••••		 	•••••	••••	•••••				
••••	 		•••••			•••••		 	•••••	•••••	•••••	• • • • • • • • •		•••••	••••
••••	 			• • • • • • • •	• • • • • • • •	•••••		 	•••••	•••••	•••••			•••••	
••••	 							 		•••••					
••••	 				•••••	• • • • • • • •	• • • • • • • • •	 	•••••	••••	•••••				
••••	 							 		•••••					
••••	 					• • • • • • • •		 		•••••					



13786

\*36AMT1120\*

[Turn over

(a)	The curved surface of a glass bowl can be modelled by rotating the curve
	$y = 3 \tan 4x$
	between $x = 0$ and $x = \frac{\pi}{16}$ through $2\pi$ radians about the <i>x</i> -axis.
	Find, in terms of $\pi$ , the maximum volume of liquid that the bowl can contain. [9]

[Turn over	
[Turn over	
	[Turn over

(b)	Find $\int x e^{2x} dx$	[7]
		••••••
		••••••
		•••••
		•••••
		•••••
		•••••
		•••••
		•••••
		••••••
		•••••
13786		

P y Learning Reserved P y Learning

2 Learning Peasantile Pasantile 2 Learning

200 J. Levaminy

Rosertin 2 Learning Rosertin

Rosertin 2 Learning Rosertin

Rosertin 2 Learning Roserting

Reserved 2 Learning Reserved Reserved

Reserved 7 Learning Reserved Reserved

Rowardin 2 Learning Rowardin

j Lawrity Research

Research 2 Learning Research

Possetion PLeasering Rosering PLeasering PLeasering

G



\*36AMT1124\*

[Turn over

\*36AMT1125\*

*36AMT1126*
-------------

	u = x - 3
	find the exact value of
	$\int_{4}^{6} \frac{x(x-5)}{(x-3)^2}  \mathrm{d}x $ [9]
13786	

(c) Using the substitution

y Learning Research 2 Loaming C. 20 7 Learning Rowards 20 7 Learning CC. 20 J Learning CC. 20 J Learning C. 20 G 200 7 Learning CC. DD 7 Learning CC. C. 20 Learning C. 200 7 Learning CC. CC. CC. Reserved Rowardin 

G

•••••••
[Turn over

\*36AMT1127\*

$2x^3 + 11x^2 + 12x - 9 $ [3]

y Learning Research

CC.



\*36AMT1128\*

(ii) Hence find

 $\int \frac{x^2 + 3x + 35}{2x^3 + 11x^2 + 12x - 9} \, \mathrm{d}x$ [15]

[Turn over	
[Turn over	
	[Turn over

\*36AMT1129\*

\*36AMT1130\*

	•••••••••••••••••••••••••••••••••••••••
13786	

••••••
••••••
[Turn over

[7]

10 In a factory, a biological substance is placed in a large tank.

During the production process, the biological substance reproduces at a rate of 0.25A kg per hour, where A kg is the amount of substance present at time t hours.

At the same time, the biological substance is pumped out of the tank at a constant rate of 50 kg per hour.

(i) By setting up and solving a suitable differential equation, show that

 $A = 4ke^{0.25t} + 200$ 

where *k* is a constant.

13786

	•••
	•••
	••
	•••
	••
	•••
	•••
	•••
	•••
	• •
	•••
	•••
	• •
	•••
	•••
	• •
	•••
	•••
	••

\*36AMT1132\*

At time t = 0, A = 190

(ii) Show that

$A = 200 - 10e^{0.25t}$	[2]
	•••••
	····· ·
	•••••
	•••••
	•••••
	•••••
	•••••
	····· ·
	••••••
	•••••
	•••••
	•••••
	•••••
	····· ·
[Tu	rn over



\*36AMT1133\*

(ii	i) Find the minimum length of time for each shift. Note that a production shift always lasts for a whole number of hours.

The factory management decides to change to 8-hour production shifts.

They plan to manage this by simply reducing the amount of biological substance in the tank at time t = 0

(iv) Find the maximum amount of biological substance at time t = 0 which makes this change feasible, giving your answer to 1 decimal place. [5]



#### THIS IS THE END OF THE QUESTION PAPER

#### **DO NOT WRITE ON THIS PAGE**

For Examiner's use only	
Question Number	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total Marks	
	•

Examiner Number

Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright holders may have been unsuccessful and CCEA will be happy to rectify any omissions of acknowledgement in future if notified.

13786/6

### 

#### \*36AMT1136\*