Mark Scheme (Results)

Summer 2017

Pearson Edexcel GCE
In Further Pure Mathematics FP1 (6667/01)
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.
   - bod – benefit of doubt
   - ft – follow through
   - the symbol \(\checkmark\) will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - \(*\) The answer is printed on the paper or ag- answer given
   - \(\Box\) or d… The second mark is dependent on gaining the first mark
4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.
General Principles for Further Pure Mathematics Marking
(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation
\[(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x = \ldots\]
\[(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x = \ldots\]

2. Formula
Attempt to use the correct formula (with values for a, b and c).

3. Completing the square
Solving \[x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 = q \pm c = 0, \quad q \neq 0, \text{ leading to } x = \ldots\]

Method marks for differentiation and integration:
1. Differentiation
Power of at least one term decreased by 1. \((x^n \rightarrow x^{n-1})\)

2. Integration
Power of at least one term increased by 1. \((x^n \rightarrow x^{n+1})\)
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f(x) = (\frac{1}{3}x^2 + \frac{4}{x^2} - 2x - 1, \ x &gt; 0)</td>
<td>Either any one of (f(6) = \text{awrt} -0.9) or (f(7) = \text{awrt} 1.4)</td>
<td>M1</td>
</tr>
<tr>
<td>f(6) = -0.88888888...</td>
<td>Sign change or (f(6) = -\text{ve}) and (f(7) = +\text{ve}) or (f(6) \times f(7) = -\text{ve}) o.e. (and (f(x)) is continuous) therefore a root / (\alpha) (exists between (x = 6) and (x = 7)) o.e.</td>
<td>A1</td>
</tr>
<tr>
<td>f(7) = 1.414965986...</td>
<td>Both (f(6) = \text{awrt} -0.9) and (f(7) = \text{awrt} 1.4), sign change and conclusion.</td>
<td></td>
</tr>
<tr>
<td>f(6) \times f(7) = -\text{ve} o.e. (and (f(x)) is continuous) therefore a root / (\alpha) (exists between (x = 6) and (x = 7)) o.e.</td>
<td>Allow (f(6) = -\frac{8}{9}) and (f(7) = \frac{208}{147}).</td>
<td></td>
</tr>
<tr>
<td>({f'(6) = 1.962962963...})</td>
<td>[2]</td>
<td></td>
</tr>
<tr>
<td>(f'(x) = \frac{2}{3}x - \frac{8}{x^3} - 2) (\frac{1}{3}x^3 \rightarrow \pm Ax) or (\frac{4}{x^2} \rightarrow \pm Bx^{-3}) or (-2x-1 \rightarrow -2)</td>
<td>At least two of these terms differentiated correctly. Correct derivative.</td>
<td>M1</td>
</tr>
<tr>
<td>(</td>
<td>f'(6) = \frac{53}{27})</td>
<td></td>
</tr>
<tr>
<td>(\alpha = 6 - \left(\frac{-0.88888888...}{1.962962963...}\right))</td>
<td>Correct application of Newton-Raphson using their values.</td>
<td>A1</td>
</tr>
<tr>
<td>(= 6.452830189...)</td>
<td>Exact form of (\alpha) is (\frac{342}{53} \approx 6.45)</td>
<td>A1 cso</td>
</tr>
<tr>
<td>(= 6.45) (2dp)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Question 1 Notes**

1. (a) **Note** Accept at least ‘sign change therefore root’ o.e. for A1.
   Any incorrect statements made in the conclusion award A0.

1. (b) **Note** Denominator in NR calculation may contain evidence for first 3 marks.
   Correct answer of 6.45 with minimal working will imply earlier marks for elements not explicitly stated. However, incorrect values leading to a correct final answer should be marked accordingly.
<table>
<thead>
<tr>
<th>Question Number</th>
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</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A = \begin{pmatrix} 2 &amp; -1 \ 4 &amp; 3 \end{pmatrix}$, $P = \begin{pmatrix} 3 &amp; 6 \ 11 &amp; -8 \end{pmatrix}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A^{-1} = \frac{1}{10} \begin{pmatrix} 3 &amp; 1 \ -4 &amp; 2 \end{pmatrix}$</td>
<td></td>
<td>M1</td>
</tr>
</tbody>
</table>

Either $\frac{1}{10}$ or $\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$

(b) $P = AB$

Way 1

$A^{-1}P = A^{-1}AB$ $\Rightarrow$ $B = A^{-1}P$

$B = \frac{1}{10} \begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix} \begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$

$= \begin{pmatrix} 2 & 1 \\ 1 & -4 \end{pmatrix}$

Multiplies their $A^{-1}$ by $P$ in correct order.
This substituted statement is sufficient.
At least 2 elements correct or $k\begin{pmatrix} 20 & 10 \\ 10 & -40 \end{pmatrix}$ oe.
May be unsimplified
Correct simplified matrix. A1

(b) $\{P = AB \Rightarrow\}$

Way 2

$\begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix} = \begin{pmatrix} 2a - c & 2b - d \\ 4a + 3c & 4b + 3d \end{pmatrix}$

$\Rightarrow a = 2, c = 1, b = 1, d = -4$

At least 2 elements are correct. A1

So, $B = \begin{pmatrix} 2 & 1 \\ 1 & -4 \end{pmatrix}$

Correct matrix. A1

5
3. (a)  

\[ x = 4t, \ y = \frac{4}{t}, \ t \neq 0 \]

\[ t = \frac{1}{4} \Rightarrow P(1, 16), \ t = 2 \Rightarrow Q(8, 2) \]

Coordinates for either \( P \) or \( Q \) are correctly stated. (Can be implied).

Finds the gradient of the chord \( PQ \) with \[ \frac{y_2 - y_1}{x_2 - x_1} \]
then uses in \[ y = -\frac{1}{m} x \].

Condone incorrect sign of gradient.

\[ m(l) = \frac{1}{2} \]

So, \( l: y = \frac{1}{2} x \) or \( 2y = x \)

(b) 

\[ xy = 16 \text{ or } y = \frac{16}{x} \text{ or } x = \frac{16}{y} \]

Correct Cartesian equation. Accept \[ \frac{4}{y} = x \text{ or } xy = 4 \] [1]

(c)  

<table>
<thead>
<tr>
<th>Way 1</th>
<th>Way 2</th>
<th>Way 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} x = \frac{16}{x} )</td>
<td>( 4 - \frac{1}{2} (4t) )</td>
<td>( 2y = \frac{16}{y} )</td>
</tr>
<tr>
<td>{ ( x^2 = 32 ) }</td>
<td>{ ( t^2 = 2 ) }</td>
<td>{ ( y^2 = 8 ) }</td>
</tr>
<tr>
<td>( (4 \sqrt{2}, \ 2 \sqrt{2}), \ (-4 \sqrt{2}, \ -2 \sqrt{2}) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Attempts to substitute their \( l \) into either their Cartesian equation or parametric equations of \( H \)

At least one set of coordinates (simplified or un-simplified) or \[ x = \pm 4 \sqrt{2}, \ y = \pm 2 \sqrt{2} \]
Both sets of simplified coordinates. Accept written in pairs as \[ x = 4 \sqrt{2}, \ y = 2 \sqrt{2} \]
\[ x = -4 \sqrt{2}, \ y = -2 \sqrt{2} \]

A1
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. (i)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) Way 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ w = \frac{p - 4i}{2 - 3i} \quad \text{arg} w = \frac{\pi}{4} ]</td>
<td>Mark (i)(a) and (i)(b) together.</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Multplies by ( \frac{(2 + 3i)}{(2 + 3i)} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>At least one of either the real or imaginary part of ( w ) is correct. Must be expanded but could be unsimplified e.g. expressed as single fraction. Condone ( a + ib ). Correct ( w ) in its simplest form.</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>( (a + ib)(2 - 3i) = (p - 4i) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 2a + 3b = p )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 3a - 2b = 4 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = \left( \frac{2p + 12}{13} \right) + \left( \frac{3p - 8}{13} \right) i )</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>( \left{ \text{arg} w = \frac{\pi}{4} \Rightarrow \right} \quad 2p + 12 = 3p - 8 \text{ o.e. seen anywhere.} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \Rightarrow p = 20 )</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>( z = (1 - \lambda i)(4 + 3i) ) and (</td>
<td>z</td>
</tr>
<tr>
<td>Way 1</td>
<td>( \sqrt{1 + \lambda^2} \sqrt{4^2 + 3^2} ) Attempts to apply (</td>
<td>(1 - \lambda i)(4 + 3i)</td>
</tr>
<tr>
<td></td>
<td>( \sqrt{1 + \lambda^2} \sqrt{4^2 + 3^2} = 45 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( { \lambda^2 = 9^2 - 1 \Rightarrow } \quad \lambda = \pm 4\sqrt{5} )</td>
<td>A1</td>
</tr>
<tr>
<td>Way 2</td>
<td>( z = (4 + 3\lambda) + (3 - 4\lambda)i ) Attempt to multiply out, group real and imaginary parts and apply the modulus.</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( \sqrt{(4 + 3\lambda)^2 + (3 - 4\lambda)^2} ) ( (4 + 3\lambda)^2 + (3 - 4\lambda)^2 = 45^2 ) or ( (4 + 3\lambda)^2 + (3 - 4\lambda)^2 = 45 ) ( {16 + 24\lambda + 9\lambda^2 + 9 - 24\lambda + 16\lambda^2 = 2025} )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>( {25\lambda^2 = 2000 \Rightarrow } \quad \lambda = \pm 4\sqrt{5} ) Condone if middle terms in expansions not explicitly stated.</td>
<td>A1</td>
</tr>
</tbody>
</table>

**Question 4 Notes**

(ii) M1 Also allow \( (1 + \lambda^2)(4^2 + 3^2) \) for M1.

M1 Also allow \( (4 + 3\lambda)^2 + (3 - 4\lambda)^2 \) for M1.
5. (i) \[
A = \begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix}, \quad B = \begin{pmatrix} -5 & 4 \\ 6 & -5 \end{pmatrix}, \quad M = \begin{pmatrix} a & -9 \\ 1 & 2 \end{pmatrix}
\]
p, a are constants.

(a) \[\{AB\} = \begin{pmatrix} -5p + 12 & 4p - 10 \\ -15 + 6p & 12 - 5p \end{pmatrix}\]

At least 2 elements are correct. M1
Correct matrix. A1

(b) \[\{AB + 2A = kI\}\]

\[
\begin{pmatrix} -5p + 12 & 4p - 10 \\ -15 + 6p & 12 - 5p \end{pmatrix} + 2 \begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix} = k \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\]

If ‘simultaneous equations’ used, allocate marks as below. M1
Forms an equation in \(p\) or eliminates \(p\) to find a value for \(k\). A1

\(4p - 10 + 4 = 0\) or \(-15 + 6p + 6 = 0\)

\(\Rightarrow p = \frac{3}{2}\)

\(k = -5 \left(\frac{3}{2}\right) + 12 + 2 \left(\frac{3}{2}\right) \Rightarrow k = \ldots\)

Substitutes their \(p = \frac{3}{2}\) into "their (-5p + 12) + 2p" to find a value for \(k\) or eliminates \(p\) to find \(k\). A1

\(k = \frac{15}{2}\) o.e

(ii) \[\pm \frac{270}{15} = \pm 18\]

Can be implied from calculations. B1

\[\det M = (a)(2) - (-9)(1)\]

Equate their \(\det A\) to either 18 or -18. M1
At least one of either \(a = 4.5\) or \(a = -13.5\). A1

Both \(a = 4.5\) and \(a = -13.5\). A1

Way 1

Consider vertices of triangle with area 15 units e.g. (0,0), (15,0) and (0,2) and attempting 2 values of \(a\).

\[\begin{pmatrix} a & -9 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 0 & 15 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 15a & -18 \\ 0 & 15 & 4 \end{pmatrix}\]

Pre-multiples their matrix by \(M\) and obtains single matrix M1
Equates their determinant to 270 and attempts to solve. M1

\[\Rightarrow a = 4.5\ or \ a = -13.5\]

At least one of either \(a = 4.5\) or \(a = -13.5\). A1
Both \(a = 4.5\) and \(a = -13.5\). A1

Way 2

Consider vertices of triangle with area 15 units e.g. (0,0), (15,0) and (0,2) and attempting 2 values of \(a\).

\[\begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 15a & -18 \\ 15 & 4 \end{pmatrix} = 270\]

Equate their determinant to 270 and attempts to solve. M1

\[\Rightarrow a = 4.5\ or \ a = -13.5\]

At least one of either \(a = 4.5\) or \(a = -13.5\). A1
Both \(a = 4.5\) and \(a = -13.5\). A1
<table>
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<tr>
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<tbody>
<tr>
<td>6.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>$x^3 + ax^2 + bx - 52 = 0, \ a, b \in \mathbb{R},$ and $2i - 3$ are roots</td>
<td>B1 [1]</td>
</tr>
<tr>
<td>(b)</td>
<td>$-2i - 3$</td>
<td></td>
</tr>
<tr>
<td><strong>Way 1</strong></td>
<td>$(x - (2i - 3))(x + &quot;(-2i - 3)&quot;'); = x^2 + 6x + 13$ or $x = -3 \pm 2i \Rightarrow (x + 3)^2 = -4$; $= x^2 + 6x + 13(= 0)$</td>
<td>M1; A1</td>
</tr>
<tr>
<td></td>
<td>$(x - 4)(x - (2i - 3)); = x^2 - (1 + 2i)x + 4(2i - 3)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(x - 4)(x - &quot;(-2i - 3)&quot;'); = x^2 - (1 - 2i)x + 4(-2i - 3)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(x - 4)(x^2 + 6x + 13) {= x^3 + ax^2 + bx - 52}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$(x - 3^{\text{rd}} \text{ root})(\text{their quadratic})$.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Could be found by comparing coefficients from long division.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$a = 2, \ b = -11$ or $x^2 + 2x^2 - 11x - 52$</td>
<td>A1</td>
</tr>
<tr>
<td><strong>Way 2</strong></td>
<td>$\text{Sum } = (2i - 3) + &quot;(-2i - 3)&quot; = -6$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\text{Product } = (2i - 3)x&quot;(-2i - 3)&quot; = 13$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>So quadratic is $x^2 + 6x + 13$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>${= x^3 + ax^2 + bx - 52}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$(x - 3^{\text{rd}} \text{ root})(\text{their quadratic})$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$a = 2, \ b = -11$ or $x^2 + 2x^2 - 11x - 52$</td>
<td>A1</td>
</tr>
<tr>
<td><strong>Way 3</strong></td>
<td>$(2i - 3)^3 + a(2i - 3)^2 + b(2i - 3) - 52 = 0$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$5a - 3b = 43 \text{ (real parts)}$ and $6a - b = 23$ (imaginary parts) or uses $f(4) = 0$ and $f(a \text{ complex root}) = 0$ to form equations in $a$ and $b.$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Substitutes $2i - 3$ into the displayed equation and equates both real and imaginary parts.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$5a - 3b = 43$ and $6a - b = 23$ or $16a + 4b = -12$ and $(2i - 3)^3 + a(2i - 3)^2 + b(2i - 3) - 52 = 0 /$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$(-2i - 3)^3 + a(-2i - 3)^2 + b(-2i - 3) - 52 = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>So $a = 2, \ b = -11$ or $x^3 + 2x^2 - 11x - 52$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Solves these equations simultaneously to find at least one of either $a = \ldots$ or $b = \ldots$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>At least one of $a = 2$ or $b = -11$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Both $a = 2$ and $b = -11$</td>
<td>A1</td>
</tr>
<tr>
<td><strong>Way 4</strong></td>
<td>$b = \text{sum of product pairs}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= 4(2i - 3) + 4&quot;(-2i - 3)&quot; + (2i - 3)&quot;(-2i - 3)&quot;$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$a = -(\text{sum of 3 roots}) = -(4 + 2i - 3&quot; - 2i - 3&quot;)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$a = 2, \ b = -11$ or $x^3 + 2x^2 - 11x - 52$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Attempts sum of product pairs.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>All pairs correct o.e.</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Adds up all 3 roots</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>At least one of $a = 2$ or $b = -11$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Both $a = 2$ and $b = -11$</td>
<td>[5]</td>
</tr>
</tbody>
</table>
Way 5

(b) Uses $f(4) = 0$

1. $16a + 4b = -12$
2. $a = -(\text{sum of 3 roots}) = -(4 + 2i - 3 - 2i - 3' - 3'')$
3. $a = 2, b = -11$ or $x^3 + 2x^2 - 11x - 52$

M1

A1

M1

A1

A1

Both $a = 2$ and $b = -11$
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y^2 = 4ax, ) at (Q(aq^2, 2aq))</td>
<td></td>
<td>M1</td>
</tr>
<tr>
<td>(y = 2\sqrt{a} x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \sqrt{a} x^{-\frac{1}{2}} ) or (2y \frac{dy}{dx} = 4a ) or (\frac{dy}{dx} = 2a \times \frac{1}{2aq} )</td>
<td>(\frac{dy}{dx} = \pm k x^{\frac{1}{2}} ) or (ky \frac{dy}{dx} = c ) or</td>
<td>A1</td>
</tr>
<tr>
<td>(\text{When } x = aq^2, \quad m_T = \frac{\sqrt{a}}{\sqrt{a} q^2} = \frac{\sqrt{a}}{a q} = \frac{1}{q} )</td>
<td>(\frac{dy}{dx} = \frac{1}{q} )</td>
<td>dM1</td>
</tr>
<tr>
<td>or when (y = 2aq, \quad m_T = \frac{4a}{2(2aq)} = \frac{1}{q} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>T:</strong> (y - 2aq = \frac{1}{q}(x - aq^2))</td>
<td>Applies (y - 2aq = (\text{their } m_T)(x - aq^2))</td>
<td>A1 * [4]</td>
</tr>
<tr>
<td><strong>T:</strong> (qy - 2aq^2 = x - aq^2)</td>
<td>or (y = (\text{their } m_T)x + c) and an attempt to find (c) with gradient from calculus.</td>
<td></td>
</tr>
<tr>
<td><strong>T:</strong> (qy = x + aq^2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>(X\left(-\frac{1}{4}a, 0\right) \Rightarrow 0 = -\frac{1}{4}a + aq^2)</td>
<td>M1</td>
</tr>
<tr>
<td>(\Rightarrow \left{q^2 = \frac{1}{4} \Rightarrow q = -\frac{1}{2} \right} \Rightarrow q = \frac{1}{2})</td>
<td>(q = \frac{1}{2})</td>
<td>A1</td>
</tr>
<tr>
<td>So, (\frac{1}{2}y = -a + a\left(\frac{1}{2}\right)^2)</td>
<td>Substitutes (x = -\frac{1}{4}a) and (y = 0) into <strong>T</strong></td>
<td>M1</td>
</tr>
<tr>
<td>giving, (y = -\frac{3a}{2}). So (D(-a, -\frac{3a}{2})) o.e.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>({\text{focus } F(a, 0)})</td>
<td>M1</td>
</tr>
<tr>
<td>Area((FXD)) = (\frac{1}{2} \left{ 5a \left(\frac{3a}{2}\right) \right} = \frac{15a^2}{16})</td>
<td>(1) ((\text{their }</td>
<td>FX</td>
</tr>
<tr>
<td>If their (y_D = \frac{1}{q}(-a + aq^2)) then require an attempt to sub for (q) to award M. (\frac{15a^2}{16}) or (0.9375a^2)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[\text{Way 1}\]
(c) **Way 2**

Area($FXD$) = \[
\begin{vmatrix}
\frac{a}{2} & -\frac{1}{4}a & -a & a \\
0 & 0 & -\frac{3}{2}a & 0
\end{vmatrix}
\]

\[= \frac{1}{2} \left[ \left( 0 + \frac{3}{8}a^2 + 0 \right) - \left( 0 + 0 - \frac{3}{2}a^2 \right) \right] = \frac{15}{16}a^2
\]

A correct attempt to apply the shoelace method.

\[
\frac{15a^2}{16} \quad \text{or} \quad 0.9375a^2
\]

M1

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[2]

(c) **Way 3**

Rectangle – triangle 1 – triangle 2

\[
= 2a \cdot \frac{3a}{2} - \frac{1}{2} \cdot \frac{3a}{2} \cdot 2 - 2 \cdot \frac{2a}{2} \cdot \frac{3a}{2} = 3a^2 - \frac{9a^2}{4} - \frac{3a^2}{2}
\]

\[
= \frac{15a^2}{16} \quad \text{or} \quad 0.9375a^2
\]

M1

A1cao

(c) **Way 4**

Attempts sine rule using appropriate choice from

\[
FX = \frac{5a}{4}, \quad FD = \frac{5a}{2}, \quad DX = \frac{3\sqrt{5}a}{4}, \quad \sin F = \frac{3}{5}, \quad \sin X = \frac{2}{\sqrt{5}}
\]

Uses Area = \[
\frac{1}{2}ab \sin C
\]

\[
\frac{15a^2}{16} \quad \text{or} \quad 0.9375a^2
\]

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**Question 7 Notes**

(c) **Way 1**

Do not award M1 if area of wrong triangle found e.g. \[
\frac{1}{2} \cdot \frac{2a}{2} = \frac{3a^2}{2}
\]
Question 8 Notes

8. (a) \[ \sum_{r=1}^{n} (3r^2 + 8r + 3) \]
\[ = \frac{3}{6} n(n+1)(2n+1) + \frac{8}{2} n(n+1) + 3n \]
\[ = \frac{1}{2} n(n+1)(2n+1) + 4n(n+1) + 3n \]
\[ = \frac{1}{2} n((2n+1)(n+1) + 8(n+1) + 6) \]
\[ = \frac{1}{2} n(2n^2 + 3n + 1 + 8n + 8 + 6) \]
\[ = \frac{1}{2} n(2n^2 + 11n + 15) \]
\[ = \frac{1}{2} n(2n+5)(n+3) \quad (*) \]

\[ \sum_{r=1}^{12} (3r^2 + 8r + 3 + k(2^{-r})) = 3520 \]

(b) \[ \sum_{r=1}^{12} (3r^2 + 8r + 3) = \frac{1}{2} (12)(29)(15) \{= 2610\} \]
\[ \sum_{r=1}^{12} (2^{-r}) = \frac{1(1-2^{12})}{1-2} \{= 4095\} \]

So, \[ 2610 + 4095k = 3520 \Rightarrow 4095k = 910 \]
giving, \[ k = \frac{2}{9} \]

\[ k = \frac{2}{9} \text{ or } 0.2 \]

**Question 8 Notes**

8. (b) **Note** 2\textsuperscript{nd} M1 1\textsuperscript{st} A1: These two marks can be implied by seeing 4095 or 4095\(k\)**
9. \( u_{n+2} = 6u_{n+1} - 9u_n \), \( n \geq 1 \), \( u_1 = 6 \), \( u_2 = 27 \); \( u_n = 3^n(n+1) \)

(i) 

If \( n = 1 \), \( u_1 = 3(2) = 6 \)

If \( n = 2 \), \( u_2 = 3^2(2+1) = 27 \)

So \( u_n \) is true when \( n = 1 \) and \( n = 2 \).

Assume that \( u_k = 3^k(k+1) \) and \( u_{k+1} = 3^{k+1}(k+2) \) are true.

Then \( u_{k+2} = 6u_{k+1} - 9u_k \)

\[ = 6^{3^{k+1}}(k+2) - 9^{3^k}(k+1) \]

\[ = 2^{3^{k+1}}(k+2) - 3^{3^{k+1}}(k+1) \]

\[ = (3^{k+1})(2k+4-k-1) \]

\[ = (3^{k+1})(k+3) \]

\[ = (3^{k+1})(k+2+1) \]

If the result is true for \( n = k \) and \( n = k+1 \) then it is now true for \( n = k+2 \). As it is true for \( n = 1 \) and \( n = 2 \) then it is true for all \( n \) \((e \mathbb{Z}^+\)).

(ii) \( f(n) = 3^{3n-2} + 2^{3n+1} \) is divisible by 19

In all ways, first \( M \) is for applying \( f(k+1) \) with at least 1 power correct. The second \( M \) is dependent on at least one accuracy being awarded and making \( f(k+1) \) the subject and the final \( A \) is correct solution only.

\( f(1) = 3^1 + 2^4 = 19 \) \{ which is divisible by 19 \}.

\{ \therefore f(n) \text{ is divisible by 19 when } n = 1 \}

\{ Assume that for \( n = k \), \}

\( f(k) = 3^{3k-2} + 2^{3k+1} \text{ is divisible by 19 for } k \in \mathbb{Z}^+. \}

\( f(k+1) - f(k) = 3^{3(k+1)-2} + 2^{3(k+1)+1} - (3^{3k-2} + 2^{3k+1}) \) \{ Applies \( f(k+1) \) with at least 1 power correct \}

\( = 7(3^{3k-2} + 2^{3k+1}) + 19(3^{3k-2}) \)

\( = 26(3^{3k-2} + 2^{3k+1}) - 19(2^{3k+1}) \)

\( = 7f(k) + 19(3^{3k-2}) \)

\( = 26f(k) - 19(2^{3k+1}) \)

\( \therefore f(k+1) = 8f(k) + 19(3^{3k-2}) \)

\( \text{or } f(k+1) = 27f(k) - 19(2^{3k+1}) \)

\{ \therefore f(k+1) = 8f(k) + 19(3^{3k-2}) \text{ is divisible by 19 as both} \}

\( 8f(k) \text{ and } 19(3^{3k-2}) \text{ are both divisible by 19} \}
If the result is true for \( n = k \), then it is now true for \( n = k + 1 \). As the result has shown to be true for \( n = 1 \), then the result is true for all \( n \) \((\in \mathbb{Z}^+)\).

Correct conclusion seen at the end. Condone true for \( n = 1 \) stated earlier.

\[ \text{Way 2} \]

(ii) \( f(1) = 3^1 + 2^1 = 19 \) \{which is divisible by 19\}.

\[ \therefore f(n) \text{ is divisible by 19 when } n = 1 \}

Assume that for \( n = k \),

\[ f(k) = 3^{3k-2} + 2^{3k+1} \]

is divisible by 19 for \( k \in \mathbb{Z}^+ \).

\[ f(k+1) = 3^{3(k+1)-2} + 2^{3(k+1)+1} \]

Applies \( f(k+1) \) with at least 1 power correct

\[ f(k+1) = 27(3^{3k-2}) + 8(2^{3k+1}) \]

\[ = 8(3^{3k-2} + 2^{3k+1}) + 19(3^{3k-2}) \]

\[ \text{either } 8(3^{3k-2} + 2^{3k+1}) \text{ or } 19(3^{3k-2}) \]

\[ \text{or } 27(3^{3k-2} + 2^{3k+1}) - 19(2^{3k+1}) \]

\[ \therefore f(k+1) = 8f(k) + 19(3^{3k-2}) \]

\[ \text{or } f(k+1) = 27f(k) - 19(2^{3k+1}) \]

\[ \{ \therefore f(k+1) = 8f(k) + 19(3^{3k-2}) \text{ is divisible by 19 as} \]

both \( 8f(k) \) and \( 19(3^{3k-2}) \) are both divisible by 19

If the result is true for \( n = k \), then it is now true for \( n = k + 1 \). As the result has shown to be true for \( n = 1 \), then the result is true for all \( n \) \((\in \mathbb{Z}^+)\).

Correct conclusion seen at the end. Condone true for \( n = 1 \) stated earlier.

\[ \text{Way 3} \]

(ii) \( f(1) = 3^1 + 2^1 = 19 \) \{which is divisible by 19\}.

\[ \therefore f(n) \text{ is divisible by 19 when } n = 1 \}

Assume that for \( n = k \),

\[ f(k) = 3^{3k-2} + 2^{3k+1} \]

is divisible by 19 for \( k \in \mathbb{Z}^+ \).

\[ f(k+1) - \alpha f(k) = 3^{3(k+1)-2} + 2^{3(k+1)+1} - \alpha(3^{3k-2} + 2^{3k+1}) \]

Applies \( f(k+1) \) with at least 1 power correct

\[ f(k+1) - \alpha f(k) = (27 - \alpha)(3^{3k-2}) + (8 - \alpha)(2^{3k+1}) \]

\[ = (8 - \alpha)(3^{3k-2} + 2^{3k+1}) + 19(3^{3k-2}) \]

\[ \text{or } (27 - \alpha)(3^{3k-2} + 2^{3k+1}) - 19(2^{3k+1}) \]

\[ \therefore f(k+1) = 8f(k) + 19(3^{3k-2}) \]

\[ \text{or } f(k+1) = 27f(k) - 19(2^{3k+1}) \]

\[ \{ \therefore f(k+1) = 27f(k) - 19(2^{3k+1}) \text{ is divisible by 19 as both} 27f(k) \]

and \( 19(2^{3k+1}) \) are both divisible by 19

If the result is true for \( n = k \), then it is now true for \( n = k + 1 \). As the result has shown to be true for \( n = 1 \), then the result is true for all \( n \) \((\in \mathbb{Z}^+)\).

Correct conclusion seen at the end. Condone true for \( n = 1 \) stated earlier.

Question 9 Notes

(ii)

Accept use of \( f(k) = 3^{3k-2} + 2^{3k+1} = 19m \) o.e. and award method and accuracy as above.