Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE
In Mechanics M3 (6679/01)
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

‘M’ marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation. e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc. The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
E.g. in a moments equation, every term must be a ‘force x distance’ term or ‘mass x distance’, if we allow them to cancel ‘g’ s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
E.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

‘A’ marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

‘B’ marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.
3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol \(\sqrt{\text{ }}\) will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5 For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6 If a candidate makes more than one attempt at any question:
   a. If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b. If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7 Ignore wrong working or incorrect statements following a correct answer.
**General Principles for Mechanics Marking**
*But note that specific mark schemes may sometimes override these general principles*

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- dM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of $g = 9.8$ should be given to 2 or 3 SF.

- Use of $g = 9.81$ should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),......then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations
  
  M(A)  Taking moments about A.

  N2L  Newton’s Second Law (Equation of Motion)

  NEL  Newton’s Experimental Law (Newton’s Law of Impact)

  HL  Hooke's Law

  SHM  Simple harmonic motion

  PCLM  Principle of conservation of linear momentum

  RHS, LHS  Right hand side, left hand side.
This is a “show that” question and candidates must make it clear that they are starting from the given information and deriving the given answer. It must be clear that the forces acting on the particle are being considered. Consequently starting from \( m \omega^2 \leq \mu mg \) (which can be obtained by working backwards from the answer) scores 0/5.

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB:</td>
<td>This is a “show that” question and candidates must make it clear that they are starting from the given information and deriving the given answer. It must be clear that the forces acting on the particle are being considered. Consequently starting from ( m \omega^2 \leq \mu mg ) (which can be obtained by working backwards from the answer) scores 0/5.</td>
<td></td>
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</tbody>
</table>

1. “\( F = ma \)” is sometimes seen. Do not penalise work that follows where \( F \) is used for friction.

\[
F \leq \mu mg \quad \text{or} \quad F = \mu mg \quad \text{or} \quad F \leq \mu R \text{ and } R = mg \quad \text{or} \quad F = \mu R \text{ and } R = mg
\]

\[
F = m \omega^2 \text{ or } F \geq m \omega^2
\]

\[
m \omega^2 \leq \mu mg
\]

\[
\omega \leq \sqrt{\frac{\mu g}{r}} \quad \ast
\]

The following notes apply whatever method the candidate has attempted.

B1: \( F \leq \mu mg \) or \( F = \mu mg \) or \( F \leq \mu R \text{ and } R = mg \) or \( F = \mu R \text{ and } R = mg \) seen. Award for any of these four statements seen.

M1: Equation of motion horizontally. Acceleration in either form. Can be given in the form of an inequality. Must include \( F \).

A1: Correct equation or inequality, with acceleration \( r \omega^2 \).

dM1: Eliminate \( F \). Must now have an inequality.

A1cso: Correct completion with no errors seen and clear notation. Candidates who work with = signs but have not specified the particle is on the point of slipping or seem to be using max friction but do not state this should not be awarded this mark.

Example 1: Here are 2 “perfect” examples. As written here they score 5/5. Same work but without reference to max friction or slipping would score 4/5.

\[
F_{\text{max}} = \mu mg \quad \text{or} \quad F_{\text{max}} = \mu R \text{ and } R = mg
\]

\[
F_{\text{max}} \geq m \omega^2
\]

\[
\mu mg \geq m \omega^2 r
\]

\[
\omega \leq \sqrt{\frac{\mu g}{r}} \quad \ast
\]

Example 2: On the point of slipping: \( F = \mu mg \) or \( F = \mu R \text{ and } R = mg \).

\[
F = m \omega^2
\]

\[
\mu mg = m \omega^2 r \implies \omega = \sqrt{\frac{\mu g}{r}}
\]

Does not slip, \( \therefore \omega \leq \sqrt{\frac{\mu g}{r}} \quad \ast \)

<table>
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<tr>
<th>Example</th>
<th>Scheme</th>
<th>Marks</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>Here are 2 “perfect” examples. As written here they score 5/5. Same work but without reference to max friction or slipping would score 4/5</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( F_{\text{max}} = \mu mg ) or ( F_{\text{max}} = \mu R \text{ and } R = mg )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( F_{\text{max}} \geq m \omega^2 )</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>( \mu mg \geq m \omega^2 r )</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>( \omega \leq \sqrt{\frac{\mu g}{r}} \quad \ast )</td>
<td>A1cso  [5]</td>
</tr>
<tr>
<td>2</td>
<td>On the point of slipping: ( F = \mu mg ) or ( F = \mu R \text{ and } R = mg )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( F = m \omega^2 )</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>( \mu mg = m \omega^2 r ) ( \implies \omega = \sqrt{\frac{\mu g}{r}} )</td>
<td>dM1A1cso [5]</td>
</tr>
<tr>
<td></td>
<td>Does not slip, ( \therefore \omega \leq \sqrt{\frac{\mu g}{r}} \quad \ast )</td>
<td>A1cso  [5]</td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
<td>Marks</td>
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<tr>
<td>----------------</td>
<td>--------</td>
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</tr>
<tr>
<td>2(a)</td>
<td>$T \cos30 = 0.5g$</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>$\text{ext} = \frac{0.9}{\cos60} - 1.2 = 0.6 \text{ m}$</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>$T = \frac{\lambda x}{l} = \frac{\lambda \times &quot;0.6&quot;}{1.2}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\frac{\lambda}{2} \times \frac{\sqrt{3}}{2} = \frac{g}{2}$</td>
<td>dM1A1</td>
</tr>
<tr>
<td></td>
<td>$\lambda = \frac{2g}{\sqrt{3}} = 11.31... = 11.3 \text{ or } 11$</td>
<td>(7)</td>
</tr>
<tr>
<td>(b)</td>
<td>$T \cos60 = 0.9m\omega^2$</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>$T \cos60 = 0.9 \times 0.5\omega^2$</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>$\frac{2g}{\sqrt{3}} \times \frac{0.6}{1.2} \times \frac{1}{2} = 0.9 \times 0.5\omega^2$</td>
<td>dM1A1</td>
</tr>
<tr>
<td></td>
<td>$\omega = 2.507... = 2.5 \text{ or } 2.51$</td>
<td>(4)</td>
</tr>
<tr>
<td></td>
<td>or $\frac{0.5g}{\cos30} \times \cos60 = 0.9 \times 0.5\omega^2$</td>
<td></td>
</tr>
</tbody>
</table>

**NB:** Here and in qu 7 penalise only once for decimal answers with more than 3 sf

(a) M1 Resolve vertically. Tension must be resolved (cos or sin allowed), weight not resolved.
A1 Correct equation

M1 Use trigonometry to calculate the extension. Must not use an erroneous 1.2 m on the vertical (Ignore it on their diagram)
A1 Correct extension

M1 Use Hooke's Law with their extension.
dM1 Eliminate $T$ and solve to $\lambda = ...$ Depends on first and third M marks above
A1 Correct answer, 2 or 3 significant figures

(b) M1 Equation of motion along the radius. $T$ must be resolved (cos or sin), acceleration in either form. $m$ or 0.5
A1 Correct equation, mass to be 0.5 here or later and acceleration $0.9\omega^2$
dM1 Eliminate $T$ by using Hooke's Law with their $\lambda$ (from (a)) or using their vertical equation from (a) and solve to $\omega = ...$ Depends on the first M in (b)
A1 Correct value for $\omega$. Must be 2 or 3 sig figs

**NB:** Full marks can be awarded in (b) if use of their $T$ (obtained from a correct use of HL) and $\lambda$ leads to the correct value.
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>3.</td>
<td></td>
<td></td>
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</tbody>
</table>

\[ mv \frac{dv}{dx} = -\frac{mgR^2}{x^2} \]

\[ \frac{1}{2}v^2 = -\int gR^2x^{-2} \, dx \]

\[ \frac{1}{2}v^2 = gR^2x^{-1} \quad (+c) \]

\[ x = 3R \quad v = \sqrt{\frac{gR}{3}} \quad \Rightarrow \quad c = \frac{1}{2} \frac{gR}{3} - \frac{gR^2}{3R} = -\frac{gR}{6} \]

\[ v = 2 \sqrt{\frac{gR}{3}} \quad 2 \frac{gR}{3} = \frac{gR^2}{x} - \frac{gR}{6} \quad x = \ldots \]

\[ x = \frac{6R}{5} \]

Dist from surface \[ \frac{6R}{5} - R = \frac{R}{5} \quad \text{oe} \]

**M1** Attempting an equation of motion with correct number of terms and acceleration \( \frac{dv}{dx} \).

Allow with minus missing. Can be given by implication if acceleration is integrated to \( \frac{1}{2}v^2 \)

**dM1** Attempting the integration of both sides of their equation. \( x^{-2} \rightarrow x^{-1} \) Depends on the first M mark.

**A1** Correct equation after correct integration. Constant of integration may be missing.

Double sign error scores A0 here.

**dM1** Substitute \( x = 3R \quad v = \sqrt{\frac{gR}{3}} \) and obtain an expression for \( c \). Depends on the first M mark providing an attempt at integration is seen. (eg \( x^{-2} \rightarrow x^{-3} \) could score M1M0A0dM1)

**dM1** Substitute \( v = 2 \sqrt{\frac{gR}{3}} \) in their expression for \( v^2 \) and solve for \( x \) Depends on the first M mark.

**A1** Correct \( x \) Double sign error scores A0 here.

**A1cso** Correct answer from completely correct working.
### ALT 1

**Definite Integration:**

\[
\frac{mv}{dx} = -\frac{mgR^2}{x^2}
\]

\[
\int \frac{mgR^2}{x^3} v \, dv = -\int_{3R}^{x} gR^2 x^{-1} \, dx
\]

\[
\left[ \frac{1}{2} v^2 \right]_{3R}^{x} = \left[ gR^2 x^{-1} \right]_{3R}^{x}
\]

\[
\frac{1}{2} \times \frac{4gR}{3} - \frac{1}{2} \times \frac{gR}{3} = \frac{gR^2}{X} - \frac{gR}{3}
\]

\[
2 \times \frac{1}{3} = \frac{R}{X} - \frac{1}{3} \times \frac{R}{X} = \frac{5}{6}
\]

\[
X = \frac{6}{5} R
\]

Dist from surface \( = \frac{6R}{5} - R = \frac{R}{5} \) oe

---

**Scheme and Marks**

- **M1**: Attempting an equation of motion with correct number of terms and acceleration \( \frac{dv}{dx} \).
  - Allow with minus missing.
- **dM1**: Attempting the integration of both sides of their equation. \( x^{-2} \rightarrow x^{-1} \) Depends on the first M mark. Limits not needed (ignore any shown)
- **A1**: Correct integration. Ignore any limits shown.
- **dM1**: Substitute correct limits. May be as shown or both sets reversed. Depends on the first M mark.
- **dM1**: Solve to \( X = ... \) Depends on the first M mark.
- **A1**: Correct \( X \)
- **A1cso**: Correct answer from completely correct working.

### ALT 2

**Energy**: Variable force, so no integration implies no marks

\[
\frac{1}{2} mv^2 - \frac{1}{2} mu^2 = \int F \, dx = \int -\frac{mgR^2}{x^2} \, dx
\]

\[
\frac{1}{2} mv^2 - \frac{1}{2} mu^2 = \left[ \frac{mgR^2}{x} \right]_{3R}^{x}
\]

\[
\frac{1}{2} \times \frac{4gR}{3} - \frac{1}{2} \times \frac{gR}{3} = \frac{mgR^2}{X} - \frac{mgR}{3}
\]

\[
X = \frac{6}{5} R, \quad \text{Dist from surface} = \frac{6R}{5} - R = \frac{R}{5} \quad \text{oe}
\]

---

**Scheme and Marks**

- **M1** (minus and limits may be missing)
- **dM1A1**: Integrate RHS (as above)
- **dM1A0**: Inconsistent signs scores
- **dM1**: Sub correct limits
- **dM1A1**, **A1cso**: As alt 1
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<th>Question Number</th>
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<tr>
<td>4</td>
<td>Elastic energy $= \frac{1}{2} \times 2mg \frac{x^2}{l}$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Work done by friction $= (l + x) \mu mg \cos \alpha$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Energy from release: $(l + x) \mu mg \cos \alpha + \frac{1}{2} \times 2mg \frac{x^2}{l} = (l + x)mg \sin \alpha$</td>
<td>M1A1 ft work and EE</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{4} \times 4(l + x) + \frac{x^2}{l} = \frac{3}{5}(l + x)$</td>
<td>dM1A1</td>
</tr>
<tr>
<td></td>
<td>$l^2 + 4lx + 5x^2 = 3l^2 + 3lx$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$5x^2 - 2lx - 2l^2 = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = 0.863...l$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$k = 1.86$</td>
<td>A1 (7) [7]</td>
</tr>
</tbody>
</table>

**B1** Correct elastic energy

**B1** Correct work done by friction

**M1** Attempt a work-energy equation. Must have 3 terms: work done by friction, elastic energy, GPE.

EPE term must be of the form $= k\lambda \frac{x^2}{l}$ $k = 1, 2$ or $\frac{1}{2}$

Work done term must be of the form distance $\times \mu mg \cos \alpha$ or $\sin \alpha$

**A1**ft Correct equation, ft their EPE and work terms

**dM1** Solve their 3 term quadratic to obtain a value for the extension as a multiple of $l$. Award if correct answer follows a correct quadratic. If the quadratic is incorrect award only if working shown (ie general formula shown explicitly and used or by implication through substitution correct for their equation, pos root only needed)

**A1** Correct extension decimal or exact

**A1** Complete by adding 1 to the numerical multiple of $l$. Must be 3 significant figures.

**ALT** Using dist moved: $kl$ or $x$

EPE $= \frac{1}{2} \times 2mg \frac{(kl-l)^2}{l}$, WD $= kl \mu mg \cos \alpha$ B1, B1

$kl \mu mg \cos \alpha + \frac{1}{2} \times 2mg \frac{(kl-l)^2}{l} = klmg \sin \alpha$ M1A1 ft

$5k^2 - 12k + 5 = 0$

$k = 1.86$ M1A2 (Give A1 for correct ans with more than 3 sf or exact)
<table>
<thead>
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<tbody>
<tr>
<td>5(a)</td>
<td>((\pi)\int_0^2 y^2 x , dx = (\pi)\int_0^2 (r^2 - x^2) x , dx)</td>
</tr>
<tr>
<td></td>
<td>(= (\pi)\left[\frac{1}{2} r^2 x^2 - \frac{1}{4} x^4\right]_0^2 = \frac{1}{2} \pi r^4 - \frac{1}{4} (\pi) r^4 \left(= \frac{1}{4} (\pi) r^4\right))</td>
</tr>
<tr>
<td></td>
<td>(\bar{x} = \frac{\int_0^2 \pi y^2 x , dx}{\frac{2}{3} \pi r^3} = \frac{3}{8} r) *</td>
</tr>
<tr>
<td></td>
<td>All marks available if (\pi) omitted throughout.</td>
</tr>
<tr>
<td>(b)</td>
<td>Mass</td>
</tr>
<tr>
<td></td>
<td>(\frac{2}{3} \pi a^3)</td>
</tr>
<tr>
<td></td>
<td>(\frac{2}{3} \pi \left(\frac{1}{2} a\right)^3 k)</td>
</tr>
<tr>
<td></td>
<td>(\frac{2}{3} \pi a^3 \left(1 + \frac{1}{k}\right))</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>(\frac{1}{8})</td>
</tr>
<tr>
<td></td>
<td>(\left(1 + \frac{1}{k}\right))</td>
</tr>
<tr>
<td></td>
<td>(NB: No penalty if (\frac{4}{3} \pi r^3) used instead of (\frac{2}{3} \pi r^3))</td>
</tr>
<tr>
<td></td>
<td>Dist from (O)</td>
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<tr>
<td></td>
<td>(\frac{3}{8} a)</td>
</tr>
<tr>
<td></td>
<td>((-\frac{3}{16} a) \bar{x})</td>
</tr>
<tr>
<td></td>
<td>(\frac{3}{8} a - \frac{3}{8 \times 16} ak = \left(1 + \frac{1}{k}\right) \bar{x})</td>
</tr>
<tr>
<td></td>
<td>(\bar{x} = \frac{\left[(48 - 3k)a\right]}{16(8 + k)}) oe Must have modulus signs</td>
</tr>
<tr>
<td>(c)</td>
<td>(\bar{x} = 0), (\therefore k = 16) ft only if (k &gt; 0)</td>
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<table>
<thead>
<tr>
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<tr>
<td>M1A1</td>
<td></td>
</tr>
<tr>
<td>dM1,A1</td>
<td></td>
</tr>
<tr>
<td>M1A1csO</td>
<td>(6)</td>
</tr>
<tr>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>M1A1ft</td>
<td>(2) [13]</td>
</tr>
</tbody>
</table>

(a)M1 Using \((\pi)\int_0^2 y^2 x \, dx\) with or without \(\pi\). Must be dimensionally correct with integrand of the form \((a^2 - x^2)x\). Limits not needed.

A1 \((\pi)\int_0^2 (r^2 - x^2)x \, dx\) Correct integral, with or without \(\pi\). Limits not needed.

dM1 Attempt the integration and include correct limits for their equation ie 0,\(r\) or 0,\(a\). Depends on the first M mark.

Correct result, seen explicitly here or at the final stage. Award for \(\frac{1}{2} \pi r^4 - \frac{1}{4} (\pi) r^4\) or

A1 \(\frac{1}{4} (\pi) r^4\)

dM1 Using \(\bar{x} = \frac{\int_0^2 \pi y^2 x \, dx}{\frac{2}{3} \pi r^3}\) \(\pi\) in numerator and denominator or in neither. Depends on 1st M

A1csO Correct given result with no errors seen.

**ALT:** Start by finding the distance of the c of m from the point of intersection of the axis of symmetry and the surface of the hemisphere:

Equation needed is \(y^2 = 2xr - x^2\) leading to distance = 5/8 \(r\)

Mark as main scheme. Score 5/6 for all correct apart from completion to 3/8 \(r\)
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)B1</td>
<td>Correct mass ratio - any equivalent</td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>Correct distances from $O$ or any other point</td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td>Attempting a dimensionally consistent moments equation.</td>
<td></td>
</tr>
<tr>
<td>A1ft</td>
<td>Correct equation, follow through their mass ratio and distances. Signs to be correct here.</td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>Correct final answer. Must have modulus signs ((\text{sign of } 48 - 3k \text{ is not known})) Numerator can be (</td>
<td>(3k - 48)\alpha</td>
</tr>
<tr>
<td>(c)M1</td>
<td>Setting their $\bar{x} = 0$ and solve to $k = ...$ (or imply this by using the separate C of Ms and the respective masses)</td>
<td></td>
</tr>
<tr>
<td>A1ft</td>
<td>Correct $k$ follow through their $\bar{x}$ (Award if earned even if no modulus signs in (b))</td>
<td></td>
</tr>
<tr>
<td>Question Number</td>
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<td>Marks</td>
</tr>
<tr>
<td>-----------------</td>
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<td>-------</td>
</tr>
<tr>
<td><strong>6(a)</strong></td>
<td>( \frac{1}{2}m \times u^2 - \frac{1}{2}m v^2 = mga \sin \theta )</td>
<td>M1A1A1</td>
</tr>
<tr>
<td></td>
<td>( T + mga \sin \theta = \frac{m v^2}{a} )</td>
<td>M1A1A1</td>
</tr>
<tr>
<td></td>
<td>( T + mga \sin \theta = \frac{mu^2}{a} - 2mga \sin \theta )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( T = \frac{m}{a} \left( u^2 - 3ga \sin \theta \right) )</td>
<td>dM1A1cs o ((8))</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>At top ( T \geq 0 ) ( \Rightarrow u^2 \geq 3ag )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( u_{\text{min}} = \sqrt{3ag} )</td>
<td>A1 ((2))</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>Least at top: ( T_{\text{least}} = \frac{m}{a} \left( u^2 - 3ag \right) ) ((= S))</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>Greatest at bottom: ( T_{\text{greatest}} = \frac{m}{a} \left( u^2 + 3ag \right) ) ((= 4S))</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(M1A1 for either A1 for second one)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 4 \times \frac{m}{a} \left( u^2 - 3ag \right) = \frac{m}{a} \left( u^2 + 3ag \right) )</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>( 4u^2 - 12ag = u^2 + 3ag )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 3u^2 = 15ag, \ u = \sqrt{5ag} )</td>
<td>A1 ((5))</td>
</tr>
</tbody>
</table>

(a) M1 Attempt an energy equation from \( A \) to the general position (as shown in the diagram). Must have a difference of two KE terms and a gain of PE (one or two terms)
A1A1-1 each error
M1 Attempt an equation of motion along the radius at the general position. Weight must be resolved (sin or cos). Acceleration can be in either form
A1 Resultant force correct
A1 Correct acceleration (as shown)
dM1 Eliminate \( v^2 \) between their two equations and solve to \( T = \ldots \) Depends on both previous M marks.
A1cs o Obtain the given expression for \( T \) with no errors seen
(b) M1 Use \( \sin \theta = 1 \) so \( T \geq 0 \) at the top. Allow with \( \geq \) or \( > \) OR State min \( u \) when \( T = 0 \) at the top
A1 \( u_{\text{min}} = \sqrt{3ag} \) \( u_{\text{min}} \geq 3ag \) or \( u_{\text{min}} \geq 3ag \) scores A0
(c) M1 Use the result given in (a) to obtain the tension at the top \((\theta = 90^\circ)\) or the tension at the bottom \((\theta = 270^\circ)\) Alt: Energy equation from \( A \) to either or both top and bottom.
A1 A1 One mark for each correct (Enter A1A1, A1A0 or A0A0)
dM1 Form an equation with \( 4 \times \) their least = their greatest Both tensions to be of the form
\( \frac{m}{a} \left( u^2 \pm kag \right) \) \( k \neq 0 \). Depends on the previous M mark
A1 Correct expression for \( u \).
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<tr>
<td>7(a)</td>
<td>0.5g = \frac{29.4(1.4 - l)}{l} OR 0.5g = \frac{29.4x}{l}</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>x = \frac{l}{6}, 7l/6 = 1.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>\quad l = 1.2 *</td>
<td>A1cso (3)</td>
</tr>
<tr>
<td>(b)</td>
<td>0.5g - T = 0.5\ddot{x}</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>0.5g - \frac{29.4(x + 0.2)}{1.2} = 0.5\ddot{x}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>\ddot{x} = -\frac{29.4}{1.2 \times 0.5} x, \dddot{x} = -49x \quad : \text{SHM}</td>
<td>dM1A1 (4)</td>
</tr>
<tr>
<td>(c)</td>
<td>v^2 = 49\left(0.4^2 - (\pm 0.2)^2\right)</td>
<td>M1A1ft</td>
</tr>
<tr>
<td></td>
<td>v = 2.42487... = 2.4 or 2.42 m/s</td>
<td>A1 (3)</td>
</tr>
<tr>
<td>(d)</td>
<td>Motion under gravity: 0 = 7 \sqrt{0.4^2 - 0.2^2} - gt</td>
<td>M1A1ft</td>
</tr>
<tr>
<td></td>
<td>t = \left(7\sqrt{0.4^2 - 0.2^2}\right) + g = 0.24743....</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>\text{SHM: } -0.2 = 0.4 \cos 7t</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>\quad t = \frac{1}{7} \cos^{-1} (-0.5) = 0.29919...</td>
<td>dM1A1</td>
</tr>
<tr>
<td></td>
<td>\text{Total time } = 0.29919... + 0.24743.... = 0.5466... = 0.55 or 0.547 s</td>
<td>A1 cao (7) [17]</td>
</tr>
</tbody>
</table>

(a) M1 Use Hooke's Law to find the extension at B
A1 Correct equation
A1 Obtain given value for \( l \) with no errors seen

(b) M1 Attempt an equation of motion, using Hooke's law for the tension when extension is \( x + 0.2 \)
m or 0.5 allowed Acceleration can be \( \ddot{x} \) or \( a \)
A1 Fully correct equation \( m \) or 0.5 allowed Acceleration can be \( \ddot{x} \) or \( a \) but if \( a \) used the direction must be consistent with the direction for \( \ddot{x} \)
dM1 Re-arrange to the form \( \ddot{x} = (\pm)\omega^2x \) Must be \( \dddot{x} \) now and probably will have 0.5 for mass.
Depends on the first M mark
A1 Correct equation and conclusion stated.
(c)  
M1 Use \( v^2 = \omega^2 (a^2 - x^2) \) with their \( \omega^2 \), obtained from a "correct" equation ie \( \ddot{x} \) or \( a = -\omega^2 x \) and \( x = \pm 0.2 \) amp = 0.4  
A1ft Correct equation, follow through their \( \omega \)  
A1 Correct speed at instant the string becomes slack.  
Must be 2 or 3 significant figures as value used for \( g \) in (a)  

NB Can be solved using energy.  
M1 Energy equation with an EPE term, a (final) KE term and a GPE term, all with the correct dimensions.  
A1 All terms correct (No follow through on this method)  
A1 Correct speed at instant the string becomes slack.  
Must be 2 or 3 significant figures as value used for \( g \) in (a)  

(d)  
M1 Use SUVAT for the motion under gravity with their speed from (c) to find the time from \( D \) to the string becoming taut again. The exact method chosen must be complete.  
A1ft Correct numbers used, follow through their speed from (c)  
A1 Correct time, shown explicitly or implied by correct final answer. Need not be 2 or 3 sf as not a demanded answer.  
M1 \( (\pm)0.2 = 0.4 \cos \theta t \quad \text{or} \quad 0.4 \sin \theta t \) with their \( \omega \) from a “correct” equation (see (c))  
dM1 Solve for the time to \( C \). Must be radians. Can be implied by a correct final answer.  
The method here must be complete. Depends on the previous M mark.  
A1 Correct time. Can be implied by a correct final answer.  
A1 Add the times for the 2 parts of the motion to obtain the correct total time. Must be 2 or 3 significant figures.