Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE Mathematics
Core Mathematics C4 (6666)
Edexcel and BTEC Qualifications

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations
   
   These are some of the traditional marking abbreviations that will appear in the mark schemes.
   
   - bod – benefit of doubt
   - ft – follow through
   - the symbol √ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - o.e. – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper
   - [ ] The second mark is dependent on gaining the first mark
   - dM1 denotes a method mark which is dependent upon the award of the previous method mark.
   - aef “any equivalent form”

4. All A marks are 'correct answer only’ (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
   • If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   • If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.
**General Principles for Core Mathematics Marking**

*(But note that specific mark schemes may sometimes override these general principles)*

**Method mark for solving 3 term quadratic:**

1. **Factorisation**

\[(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x = \ldots\]

\[(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x = \ldots\]

2. **Formula**

Attempt to use the correct formula (with values for \(a\), \(b\) and \(c\)).

3. **Completing the square**

Solving \(x^2 + bx + c = 0\) : \(\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \text{ leading to } x = \ldots\)

**Method marks for differentiation and integration:**

1. **Differentiation**

Power of at least one term decreased by 1. \((x^n \to x^{n-1})\)

2. **Integration**

Power of at least one term increased by 1. \((x^n \to x^{n+1})\)
Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners’ reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners’ reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Notes</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (a)</td>
<td></td>
<td></td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$\sqrt{(4 - 9x)} = (4 - 9x)^{\frac{1}{2}} = (4)^{\frac{1}{2}} - (9x)^{\frac{1}{2}} = 2\left(1 - \frac{9x}{4}\right)^{\frac{1}{2}}$</td>
<td>E.g. For $10\sqrt{3.1}$ (can be implied by later working) and $x = 0.1$ (or uses $x = 0.1$)</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$= \left{2\right} \left[1 + \left(\frac{1}{2}\right)(kx) + \frac{\left(\frac{1}{2}\right)(\frac{1}{2})(kx)^2}{2!} + \ldots\right]$</td>
<td>Note: $\sqrt{(100)(3.1)}$ by itself is B0</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= \left{2\right} \left[1 + \left(\frac{1}{2}\right)\left(\frac{9x}{4}\right) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{2!}\left(\frac{9x}{4}\right)^2 + \ldots\right]$</td>
<td>Substitutes their $x$, where $</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>$= 2\left[1 - \frac{9}{8}x - \frac{81}{128}x^2 + \ldots\right]$</td>
<td>So, $\sqrt{310} \approx 17.6234375$</td>
<td>17.623 cao</td>
</tr>
<tr>
<td></td>
<td>$= 2 - 0.225 - 0.01265625 = 1.76234375$</td>
<td>Note: the calculator value of $\sqrt{310}$ is $17.60681686...$ which is $17.607$ to 3 decimal places</td>
<td>[3]</td>
</tr>
<tr>
<td></td>
<td>$\sqrt{310} \approx 17.6234375$</td>
<td>Note:</td>
<td>8 marks</td>
</tr>
</tbody>
</table>

### Question 1 Notes

1. (a) **B1** (4)$^{\frac{1}{2}}$ or 2 outside brackets or 2 as candidate’s constant term in their binomial expansion

   **M1** Expands $(... + kx)^{\frac{1}{2}}$ to give any 2 terms out of 3 terms simplified or un-simplified,

   E.g. $1 + \left(\frac{1}{2}\right)(kx)$ or $\left(\frac{1}{2}\right)(kx) + \frac{\left(\frac{1}{2}\right)(-\frac{1}{2})(kx)^2}{2!}$ or $1 + ... + \frac{\left(\frac{1}{2}\right)(-\frac{1}{2})}{2!}(kx)^2$

   where $k$ is a numerical value and **where** $k \neq 1$

   **A1ft** A correct simplified or un-simplified $1 + \left(\frac{1}{2}\right)(kx) + \frac{\left(\frac{1}{2}\right)(-\frac{1}{2})}{2!}(kx)^2$ expansion with **consistent** $(kx)$

   **Note** $(kx), k \neq 1$ must be consistent (on the RHS, not necessarily on the LHS) in their expansion

   **Note** Award B1M1A0 for $2\left[1 + \left(\frac{1}{2}\right)\left(\frac{9x}{4}\right) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{2!}\left(\frac{9x}{4}\right)^2 + \ldots\right]$ because $(kx)$ is not consistent

   **Note** **Incorrect bracketing:** $2\left[1 + \left(\frac{1}{2}\right)\left(\frac{9x}{4}\right) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{2!}\left(\frac{9x^2}{4}\right)^2 + \ldots\right]$ is B1M1A0 unless recovered

   **A1** $2\left(\frac{9}{4}x \right. \text{(simplified fractions)} \text{ or allow} 2 \text{ } 2.25x \text{ or } 2 \text{ } 2\frac{1}{4}x$

   **A1** Accept only $\frac{81}{64}x^2$ or $-\frac{17}{64}x^2$ or $1.265625x^2$
### Question 1 Notes Continued

1. (a) SC
   - If a candidate would otherwise score 2nd A0, 3rd A0 (i.e. scores A0A0 in the final two marks to (a)) then **allow Special Case 2nd A1 for either**

     - **SC:** \[2 \left(1 - \frac{9}{8} x \ldots \right)\]  or  **SC:** \[2 \left(1 + \ldots - \frac{81}{128} x^2 \ldots \right)\]  or  **SC:** \[\lambda \left(1 - \frac{9}{8} x - \frac{81}{128} x^2 \ldots \right)\] (where \(\lambda\) can be 1 or omitted), where each term in the \([\ldots]\) is a simplified fraction or a decimal,

   **OR SC:** for \(\frac{18}{8} x \quad \frac{162}{128} x^2 + \ldots\) (i.e. for not simplifying their correct coefficients)

   - Candidates who write \[2 \left(1 + \left(\frac{9x}{4}\right) + \left(\frac{\frac{1}{2}(\frac{1}{2})}{2!}\left(\frac{9x}{4}\right)^2 \ldots \right)\] where \(k = \frac{9}{4}\) and not \(\frac{9}{4}\) and achieve \(2 + \frac{9}{4} x; \frac{81}{64} x^2 \ldots\) will get B1M1A1A1A1

   - Note Ignore extra terms beyond the term in \(x^2\)

   - Note You can ignore subsequent working following a correct answer

   - Note Allow B1M1A1A1 for \(2 + \frac{9}{4} x; \frac{81}{64} x^2 \ldots\)

(b) Note Give B1 M1 for \(\sqrt{310} \approx 10 \left(2 - \frac{9}{4} (0.1) - \frac{81}{64} (0.1)^2\right)\)

   - **Note** Other alternative suitable values for \(x\) for \(\sqrt{310} \approx \beta\sqrt{4} - 9\) (their \(x\))

<table>
<thead>
<tr>
<th>(x)</th>
<th>Estimate</th>
<th>(x)</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 (\frac{38}{147})</td>
<td>17.479</td>
<td>14 (\frac{79}{294})</td>
<td>18.256</td>
</tr>
<tr>
<td>8 (\frac{3}{32})</td>
<td>17.599</td>
<td>15 (\frac{118}{405})</td>
<td>18.555</td>
</tr>
<tr>
<td>9 (\frac{14}{729})</td>
<td>17.607</td>
<td>16 (\frac{119}{384})</td>
<td>18.899</td>
</tr>
<tr>
<td>10 (\frac{1}{10})</td>
<td>17.623</td>
<td>17 (\frac{94}{289})</td>
<td>19.283</td>
</tr>
<tr>
<td>11 (\frac{58}{363})</td>
<td>17.690</td>
<td>18 (\frac{493}{1458})</td>
<td>19.701</td>
</tr>
<tr>
<td>12 (\frac{133}{648})</td>
<td>17.819</td>
<td>19 (\frac{126}{361})</td>
<td>20.150</td>
</tr>
<tr>
<td>13 (\frac{122}{507})</td>
<td>18.009</td>
<td>20 (\frac{43}{120})</td>
<td>20.625</td>
</tr>
</tbody>
</table>

Note Apply the scheme in the same way for their \(\beta\) and their \(x\)

- E.g. Give B1 M1 A1 for \(\sqrt{310} \approx 12 \left(2 - \frac{9}{4} \frac{133}{648} - \frac{81}{64} \frac{133}{648}^2\right) = 17.819\) (3 dp)

- **Note** Allow B1 M1 A1 for \(\sqrt{310} \approx 10 \left(2 - \frac{9}{4} \frac{0.441}{0.441} - \frac{81}{64} (0.441)^2\right) = 76.161\) (3 dp)

- **Note** Give B1 M1 A0 for \(\sqrt{310} \approx 10 \left(2 - \frac{9}{4} (0.1) - \frac{81}{64} (0.1)^2 \frac{729}{512} (0.1)^3\right) = 17.609\) (3 dp)
### Question 1 Notes Continued

#### 1. (b)

| Note | Send to review using $\beta = \sqrt{155}$ and $x = \frac{2}{9}$ (which gives 17.897 (3 dp)) |

| Note | Send to review using $\beta = \sqrt{1000}$ and $x = 0.41$ (which gives 27.346 (3 dp)) |

#### 1. (a) Alternative method 1:

Candidates can apply an alternative form of the binomial expansion

$$(4 - 9x)^{\frac{1}{2}} = (4)^{\frac{1}{2}} + \binom{\frac{1}{2}}{1}(4)^{-\frac{1}{2}}(-9x) + \binom{\frac{1}{2}}{2}(4)^{-\frac{3}{2}}(-9x)^2$$

| B1 | $(4)^{\frac{1}{2}}$ or 2 |
| M1 | Any two of three (un-simplified) terms correct |
| A1 | All three (un-simplified) terms correct |
| A1 | $2 \cdot \frac{9}{4}x$ (simplified fractions) or allow $2 \cdot 2.25x$ or $2 \cdot \frac{1}{4}x$ |

| A1 | Accept only $\frac{81}{64}x^2$ or $-\frac{17}{64}x^2$ or $1.265625x^2$ |

**Note**
The terms in $C$ need to be evaluated.

So, $\frac{1}{2}C_0(4)^{\frac{1}{2}} + \frac{1}{2}C_1(4)^{-\frac{1}{2}}(-9x); + \frac{1}{2}C_2(4)^{-\frac{3}{2}}(-9x)^2$ without further working is B0M0A0

#### 1. (a) Alternative Method 2: Maclaurin Expansion $f(x) = (4 - 9x)^{\frac{1}{2}}$

| $f''(x) = -\frac{81}{4}(4 - 9x)^{-\frac{3}{2}}$ | Correct $f''(x)$ |
| $f'(x) = \frac{1}{2}(4 - 9x)^{-\frac{1}{2}}(-9)$ | $\pm a(4 - 9x)^{-\frac{1}{2}}$; $a \neq \pm 1$ |

$\therefore f(0) = 2$, $f'(0) = -\frac{9}{4}$ and $f''(0) = -\frac{81}{32}$

So, $f(x) = 2 \cdot \frac{9}{4}x; \frac{81}{64}x^2 + ...$
<table>
<thead>
<tr>
<th>Question Number</th>
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<th>Notes</th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>2.</strong></td>
<td>$x^2 + xy + y^2 = 4x + 5y + 1 = 0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| (a)             | \[
\left\{ \begin{align*}
2x + y + (x + 2y) \frac{dy}{dx} - 4 - 5 \frac{dy}{dx} &= 0 \\
2x + y &= 4 + (x + 2y) \\
\frac{dy}{dx} &= \frac{2x + y}{5} \quad \text{or} \quad \frac{4}{2x + y} \quad 5
\end{align*} \right. \\
\] | M1A1 A1 | |
| (b)             | \[
\left\{ \begin{align*}
\frac{dy}{dx} &= 0 \Rightarrow 2x + y - 4 = 0 \\
\frac{dy}{dx} &= \frac{x}{y} \quad \text{or} \quad \frac{4}{2x + y} \quad 5
\end{align*} \right. \\
\] | M1 dM1 A1 cso | |
| **(b) Alt 1**   | \[
\left\{ \begin{align*}
\frac{dy}{dx} &= 0 \Rightarrow 2x + y - 4 = 0 \\
\frac{dy}{dx} &= \frac{x}{y} \quad \text{or} \quad \frac{4}{2x + y} \quad 5
\end{align*} \right. \\
\] | M1 dM1 A1 cso | |
| (a) Alt 1       | \[
\left\{ \begin{align*}
2x \frac{dx}{dy} + \left( y \frac{dx}{dy} + x \right) + 2y - 4 \frac{dx}{dy} - 5 &= 0 \\
x + 2y - 5 + (2x + y - 4) \frac{dx}{dy} &= 0 \\
\frac{dx}{dy} &= \frac{2x + y}{5} \quad \text{or} \quad \frac{4}{2x + y} \quad 5
\end{align*} \right. \\
\] | M1A1 B1 dM1 A1 cso | |
### Question 2 Notes

**2. (a)**

<table>
<thead>
<tr>
<th>M1</th>
<th>Differentiates implicitly to include either ( \frac{dy}{dx} ) or ( y^2 \rightarrow 2y \frac{dy}{dx} ) or ( 5y \rightarrow 5 \frac{dy}{dx} ). Ignore ( \frac{dy}{dx} = \ldots )</th>
</tr>
</thead>
</table>

- A1: \( x^2 \rightarrow 2x \) and \( y^2 \) \( 4x \) \( 5y + 1 = 0 \rightarrow 2y \frac{dy}{dx} \) \( 4 \) \( 5 \frac{dy}{dx} = 0 \)
- B1: \( xy \rightarrow y + x \frac{dy}{dx} \)

**Notes**
- If an extra term appears then award 1st A0
- 2x + y + x \( \frac{dy}{dx} \) + 2y \( \frac{dy}{dx} \) \( 4 \) \( 5 \frac{dy}{dx} = 2x + y \) \( 4 = \frac{dy}{dx} \) \( 2y \frac{dy}{dx} + 5 \frac{dy}{dx} \)

**dM1** dependent on the previous M mark

An attempt to factorise out all the terms in \( \frac{dy}{dx} \) as long as there are at least two terms in \( \frac{dy}{dx} \).

- A1 \( \frac{2x + y}{5x} \) or \( \frac{4x}{2y} \) or \( \frac{2x}{x + 2y} \) or \( \frac{y}{5} \)
- cso: If the candidate’s solution is not completely correct, then do not give the final A mark

**2. (b)**

<table>
<thead>
<tr>
<th>M1</th>
<th>Sets the numerator of their ( \frac{dy}{dx} ) equal to zero (or the denominator of their ( \frac{dx}{dy} ) equal to zero) o.e.</th>
</tr>
</thead>
</table>

**Note**
- This mark can also be gained by setting \( \frac{dy}{dx} \) equal to zero in their differentiated equation from (a)
- If the numerator involves one variable only then only the 1st M1 mark is possible in part (b).

**dM1** dependent on the previous M mark

Substitutes their \( x \) or their \( y \) (from their numerator = 0) into the printed equation to give an equation in one variable only

- A1
  - For obtaining the correct 3TQ: E.g.: either \( 3x^2 - 6x - 3 = 0 \) or \( -3x^2 + 6x + 3 = 0 \)
  - This mark can also be awarded for a correct 3 term equation. E.g. either \( 3x^2 \) \( 6x = 3 \)
  - \( x^2 \) \( 2x = 1 = 0 \) or \( x^2 = 2x + 1 \) are all fine for A1

**ddM1** dependent on the previous 2 M marks

See page 6: Method mark for solving THEIR 3-term quadratic in one variable

**Quadratic Equation to solve:** \( 3x^2 - 6x = 3 = 0 \)

**Way 1:**
\[
x = \frac{6 \pm \sqrt{(6)^2 - 4(3)(3)}}{2(3)}
\]

**Way 2:** \( x^2 - 2x - 1 = 0 \) \( (x - 1)^2 - 1 - 1 = 0 \) \( x = \ldots \)

**Way 3:** Or writes down at least one exact correct \( x \)-root (or one correct \( x \)-root to 2 dp) from their quadratic equation. This is usually found on their calculator.

**Way 4:** (Only allowed if their 3TQ can be factorised)
- \( (x^2 + bx + c) = (x + p)(x + q) \), where \( |pq| = |c| \), leading to \( x = \ldots \)
- \( (ax^2 + bx + c) = (mx + p)(nx + q) \), where \( |pq| = |c| \) and \( |mn| = a \), leading to \( x = \ldots \)

**Note**
- If a candidate applies the alternative method then they also need to use their \( x = \frac{4y}{2} \) to find at least one value for \( x \) in order to gain the final M mark.

- A1: Exact values of \( x = 1 + \sqrt{2}, 1 - \sqrt{2} \) (or 1 ± \( \sqrt{2} \)), cso: Apply isw if \( y \)-values are also found.

**Note**
- It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for \( \frac{dy}{dx} \)) to gain all 5 marks in part (b)
### Question 2 Notes

**2. (a)**

#### M1
Differentiates implicitly to include either \( y \frac{dx}{dy} \) or \( x^2 \rightarrow 2x \frac{dx}{dy} \) or \( -4x \rightarrow -4 \frac{dx}{dy} \). (Ignore \( \frac{dy}{dx} = ... \))

#### A1
\[ \frac{dx}{dy} + \frac{dy}{dx} + x = 2y = 4 \]
\[ \frac{dx}{dy} + \frac{dy}{dx} + 4 \]

#### B1
\[ xy \rightarrow y \frac{dx}{dy} + x \]

**Note**
If an extra term appears then award 1\(^{st}\) A0

**Note**
\[ 2x \frac{dx}{dy} + y \frac{dx}{dy} + x + 2y - 4 \frac{dx}{dy} - 5 \rightarrow x + 2y - 5 = -2x \frac{dx}{dy} - y \frac{dx}{dy} + 4 \frac{dx}{dy} \]
will get 1\(^{st}\) A1 (implied) as the "= 0" can be implied the rearrangement of their equation.

#### dM1
**dependent on the previous M mark**
An attempt to factorise out all the terms in \( \frac{dx}{dy} \) as long as there are at least two terms in \( \frac{dx}{dy} \)

#### A1
\[ \frac{dy}{dx} = 2x + y - 4 \text{ or } \frac{dy}{dx} = 4 - 2x - y \]
\[ \frac{dx}{5 - x - 2y} \]
\[ \frac{dx}{x + 2y - 5} \]

**cso**
If the candidate’s solution is not completely correct, then do not give the final A mark

#### (a)
**Note**
Writing down from no working
- \( \frac{dy}{dx} = 2x + y - 4 \text{ or } \frac{dy}{dx} = 4 - 2x - y \) scores M1 A1 B1 M1 A1
- \( \frac{dy}{dx} = 4 - 2x - y \text{ or } \frac{dy}{dx} = 2x + y - 4 \) scores M1 A0 B1 M1 A0

**Note**
Writing \( 2xdx + ydx + xdy + 2ydy - 4dx - 5dy = 0 \) scores M1 A1 B1
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Notes</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. (i)</td>
<td>[ \frac{13 - 4x}{(2x+1)^2(x+3)} = \frac{A}{(2x+1)} + \frac{B}{(2x+1)^2} + \frac{C}{(x+3)} ]</td>
<td>At least one of ( B = 6 ) or ( C = 1 )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( B = 6, \ C = 1 )</td>
<td>Both ( B = 6 ) and ( C = 1 )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( 13 - 4x = A(2x+1)(x+3) + B(x+3) + C(2x+1)^2 )</td>
<td>Writes down a correct identity and attempts to find the value of either one of ( A ) or ( B ) or ( C )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( x = -3 \Rightarrow 25 = 25C \Rightarrow C = 1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( x = -\frac{1}{2} \Rightarrow 13 - \frac{5}{2}B \Rightarrow 15 = 2.5B \Rightarrow B = 6 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Either ( x^2 : 0 = 2A + 4C ), constant : ( 13 = 3A + 3B + C ), ( x : 4 = 7A + B + 4C ) or ( x = 0 \Rightarrow 13 = 3A + 3B + C ) leading to ( A = 2 )</td>
<td>Using a correct identity to find ( A = 2 )</td>
<td>A1</td>
</tr>
<tr>
<td>(a)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>[ \int \frac{13 - 4x}{(2x+1)^2(x+3)} , dx = \int \frac{-2}{(2x+1)} + \frac{6}{(2x+1)^2} + \frac{1}{(x+3)} , dx ]</td>
<td>( \text{At least two terms correctly integrated} )</td>
<td>A1ft</td>
</tr>
<tr>
<td></td>
<td>[ = (-2) \ln(2x+1) + \frac{6(2x+1)^{-1}}{(-1)(2)} + \ln(x+3) { + c } ]</td>
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<tr>
<td></td>
<td>( \text{o.e.} { = -\ln(2x+1) - 3(2x+1)^{-1} + \ln(x+3) { + c } } ]</td>
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<tr>
<td>(ii)</td>
<td>( { e^x + 1 }^3 = e^{3x} + 3e^{2x} + 3e^x + 1 )</td>
<td>( e^{3x} + 3e^{2x} + 3e^x + 1 ), simplified or un-simplified</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( \left{ \int (e^x + 1)^3 , dx \right} = \frac{1}{3} e^{3x} + \frac{3}{2} e^{2x} + 3e^x + x { + c } )</td>
<td>( \text{At least 3 examples (see notes) of correct ft integration} )</td>
<td>M1</td>
</tr>
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<td></td>
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<tr>
<td>(iii)</td>
<td>( \int \frac{1}{4x + 5x^2} , dx, \ x &gt; 0; \ u^3 = x )</td>
<td>( 3u^2 \frac{du}{dx} = 1 ) or ( \frac{dx}{du} = 3u^2 ) or ( \frac{du}{dx} = \frac{1}{3} x^{\frac{2}{3}} ) or ( 3u^2 du = dx ) o.e.</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>[ = \int \frac{1}{4u^3 + 5u} \cdot 3u^2 , du \left{ = \int \frac{3u}{4u^2 + 5} , du \right} ]</td>
<td>Expression of the form ( \int \frac{\pm ku^2}{4u^4 + 5u} , du ), ( k \neq 0 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( \text{Does not have to include integral sign or } du ) \hfill Can be implied by later working</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ = \frac{3}{8} \ln(4u^2 + 5) { + c } ]</td>
<td>( \text{dependent on the previous M mark} )</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>[ = \frac{3}{8} \ln \left( 4x^3 + 5 \right) { + c } ]</td>
<td>Correct answer in ( x ) with or without ( + c )</td>
<td>A1</td>
</tr>
</tbody>
</table>
### Question 3 Notes

#### Alternative method 1 for part (iii)

\[
\int \frac{1}{4x^4 + 5x^2} \, dx = \int \frac{x^{-4}}{4x^2 + 5} \, dx
\]

\[
= \frac{3}{8} \ln \left( \frac{2}{4x^2 + 5} \right) + c
\]

Attempts to multiply numerator and denominator by \( x^{-4} \) does not have to include integral sign or \( du \)

#### (i) (a)

- **M1**
  - At least 2 of either \( \pm \frac{P}{(2x+1)} \rightarrow \pm D \ln(2x+1) + \pm D \ln(x+\frac{1}{2}) \) or \( \pm \frac{Q}{(2x+1)^2} \rightarrow \pm E(2x+1) \)
  - or \( \pm \frac{R}{(x+3)} \rightarrow \pm F \ln(x+3) \) for their constants \( P, Q, R \).

- **A1ft**
  - At least 2 terms from any of \( \pm \frac{P}{(2x+1)} \) or \( \pm \frac{Q}{(2x+1)^2} \) or \( \pm \frac{R}{(x+3)} \) correctly integrated.

- **Note**
  - Can be un-simplified for the A1ft mark.

- **A1**
  - Correct answer of \( \left( \frac{3}{2} \right) \ln(2x+1) + \frac{6(2x+1)^{1/2}}{(1)(2)} + \ln(x+3) \left\{ + c \right\} \) simplified or un-simplified with or without ‘\(+c\)’.

- **Note**
  - Allow final A1 for equivalent answers, e.g. \( \ln\left( \frac{x+3}{2x+1} \right) - \frac{3}{2x+1} \left\{ + c \right\} \) or \( \ln\left( \frac{2x+6}{2x+1} \right) - \frac{3}{2x+1} \left\{ + c \right\} \)

- **Note**
  - Beware that \( \int \left( \frac{-2}{2x+1} \right) \, dx = \int \left( \frac{-1}{x+\frac{1}{2}} \right) \, dx = -\ln(x+\frac{1}{2}) \left\{ + c \right\} \) is correct integration

- **Note**
  - E.g. Allow M1 A1ft A1 for a correct un-simplified \( \ln(x+3) - \ln(x+\frac{1}{2}) - \frac{3}{2} (x+\frac{1}{2})^{-1} \left\{ + c \right\} \)

- **Note**
  - Condone 1st A1ft for poor bracketing, but do not allow poor bracketing for the final A1

- **Note**
  - E.g. Final A0 for \( -\ln(2x+1 - 3(2x+1)^{-1}) + \ln(x+3) \left\{ + c \right\} \) unless recovered

- **Note**
  - Give B1 for an un-simplified \( e^{3x} + 2e^{2x} + e^{3x} + 2e^x + e^x + 1 \)

- **M1**
  - At least 3 of either \( e^{3x} \rightarrow e^{3x} \) or \( e^{2x} \rightarrow e^{2x} \) or \( e^x \rightarrow e^x \) or \( \mu \rightarrow \mu x; \alpha, \beta, \delta, \mu \neq 0 \)

- **Note**
  - Give A1 for an un-simplified \( \frac{1}{3} e^{3x} + e^{2x} + \frac{1}{2} e^{3x} + 2e^x + e^x + x \), with or without ‘\(+c\)’

- **(iii)**
  - 1st M1 can be implied by \( \int \frac{\pm ku}{4u^2 \pm 5} \{ du \}, k \neq 0 \). Does not have to include integral sign or \( du \)

- **Note**
  - Condone 1st M1 for expressions of the form \( \int \left( \frac{\pm 1 / 4u^2 \pm 5}{u^2 \pm 5} \right) \{ du \}, k \neq 0 \)

- **Note**
  - Give 2nd M0 for \( \frac{3u}{8u} \ln(4u^2 + 5) \left\{ + c \right\} \) (\( u^2 \) not cancelled) unless recovered in later working

- **Note**
  - E.g. Give 2nd M0 for integration leading to \( \frac{3}{4} u \ln(4u^2 + 5) \) as this is not in the form \( \pm \lambda \ln(4u^2 + 5) \)
### Question Number | Scheme | Notes | Marks
---|---|---|---
3. (ii) | \[ \int (e^x + 1)^3 \, dx; \quad u = e^x + 1 \Rightarrow \frac{du}{dx} = e^x \] | \[ = \int \left( \frac{u^3}{u - 1} \right) du = \int \left( u^2 + u + 1 + \frac{1}{u - 1} \right) du \] | B1

At least 3 of either \( \alpha u^2 \rightarrow \frac{\alpha}{3} u^3 \) or \( \beta u \rightarrow \frac{\beta}{2} u^2 \) or \( \delta \rightarrow \delta u \) or \( \frac{\lambda}{u} \rightarrow \lambda \ln(u - 1); \alpha, \beta, \delta, \lambda \neq 0 \) | M1

\[ = \frac{1}{3} u^3 + \frac{1}{2} u^2 + u + \ln(u - 1) \{ + c \} \] | \[ \frac{1}{3} (e^x + 1)^3 + \frac{1}{2} (e^x + 1)^2 + (e^x + 1) + x \] or \[ \frac{1}{3} (e^x + 1)^3 + \frac{1}{2} (e^x + 1)^2 + e^x + x \] simplified or un-simplified with or without \( + c \) Note: \( \ln(e^x + 1 - 1) \) needs to be simplified to \( x \) for this mark | A1

\[ = \frac{1}{3} (e^x + 1)^3 + \frac{1}{2} (e^x + 1)^2 + (e^x + 1) + x \{ + c \} \] | \[ = \frac{1}{3} (e^x + 1)^3 + \frac{1}{2} (e^x + 1)^2 + e^x \] or \[ = \frac{1}{3} (e^x + 1)^3 + \frac{1}{2} (e^x + 1)^2 + e^x + x \] simplified or un-simplified with or without \( + c \) | A1

3. (ii) | \[ \int (e^x + 1)^3 \, dx; \quad u = e^x \Rightarrow \frac{du}{dx} = e^x \] | \[ = \int \left( \frac{(u + 1)^3}{u} \right) du = \int \left( u^2 + 3u + 3 + \frac{1}{u} \right) du \] | B1

At least 3 of either \( \alpha u^2 \rightarrow \frac{\alpha}{3} u^3 \) or \( \beta u \rightarrow \frac{\beta}{2} u^2 \) or \( \delta \rightarrow \delta u \) or \( \frac{\lambda}{u} \rightarrow \lambda \ln(u); \alpha, \beta, \delta, \lambda \neq 0 \) | M1

\[ = \frac{1}{3} u^3 + \frac{3}{2} u^2 + 3u + \ln u \{ + c \} \] | \[ = \frac{1}{3} e^{3x} + \frac{3}{2} e^{2x} + 3e^x + x \] simplified or un-simplified with or without \( + c \) Note: \( \ln(e^x) \) needs to be simplified to \( x \) for this mark | A1

\[ = \frac{1}{3} e^{3x} + \frac{3}{2} e^{2x} + 3e^x + x \{ + c \} \] | \[ = \frac{1}{3} e^{3x} + \frac{3}{2} e^{2x} + 3e^x + x \] simplified or un-simplified with or without \( + c \) | A1

[3]
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Notes</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. (a)</td>
<td>[ r = \tan 30 \implies r = h \tan 30 { \implies r = \frac{h}{\sqrt{3}} \text{ or } r = \frac{\sqrt{3}}{3} h } ]</td>
<td>Correct use of trigonometry to find ( r ) in terms of ( h ) or correct use of Pythagoras to find ( r^2 ) in terms of ( h^2 )</td>
<td>M1</td>
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<tr>
<td></td>
<td>[ h = \tan 60 \implies r = \frac{h}{\tan 60} { \implies r = \frac{h}{\sqrt{3}} \text{ or } r = \frac{\sqrt{3}}{3} h } ]</td>
<td>[ { V = \frac{1}{3} \pi r^2 h \implies V = \frac{1}{3} \pi \left( \frac{h}{\sqrt{3}} \right)^2 h \implies V = \frac{1}{9} \pi h^3 } ]</td>
<td>A1 *</td>
</tr>
<tr>
<td></td>
<td>[ h^2 + r^2 = (2r)^2 \implies r^2 = \frac{1}{3} h^2 ]</td>
<td>Or shows ( \frac{1}{9} \pi h^3 ) or ( \frac{1}{9} h^3 \pi ) with some reference to ( V = ) in their solution</td>
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<td></td>
<td>[ \frac{dV}{dt} = 200 ]</td>
<td>( \frac{dV}{dt} = \frac{1}{3} \pi h^2 \text{ o.e.} )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>[ \frac{dV}{dh} = \frac{1}{3} \pi h^2 ]</td>
<td>Either ( \left( \frac{dV}{dh} \times \frac{dh}{dr} = \frac{dV}{dr} \right) } \left( \frac{1}{3} \pi h^2 \right) \frac{dh}{dr} = 200 ]</td>
<td>M1</td>
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<td></td>
<td>either ( \left( \frac{dV}{dh} \right) \frac{dh}{dr} = 200 )</td>
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<td></td>
<td>or ( 200 \left( \frac{dV}{dh} \right) )</td>
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<td></td>
<td></td>
<td>[ \text{dependent on the previous M mark} ]</td>
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<tr>
<td></td>
<td>[ h = 15, \frac{dh}{dt} = 200 \times \frac{1}{3} \pi (15)^2 { = \frac{200}{75\pi} = \frac{600}{225\pi} } ]</td>
<td>( \frac{dh}{dr} = \frac{8}{3} \text{ (cms}^{-1}) )</td>
<td>A1 cao</td>
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| (b) Way 1 | \[ \frac{dV}{dt} = 200 \implies V = 200t + c \implies \frac{1}{9} \pi h^3 = 200t + c \] | \( \frac{dV}{dt} = \frac{1}{3} \pi h^2 \text{ o.e.} \) as in Way 1 | B1 |
|           | \( \left( \frac{1}{3} \pi h^2 \right) \frac{dh}{dr} = 200 \) | \( \text{dependent on the previous M mark} \) | M1 |
|           | \[ h = 15, \frac{dh}{dr} = 200 \times \frac{1}{3} \pi (15)^2 \{ = \frac{200}{75\pi} = \frac{600}{225\pi} \} \] | \( \frac{dh}{dr} = \frac{8}{3} \text{ (cms}^{-1}) \) | A1 cao |

| (b) Way 2 | \[ \frac{dV}{dt} = 200 \implies V = 200t + c \implies \frac{1}{9} \pi h^3 = 200t + c \] | \( \text{dependent on the previous M mark} \) | dM1 |
|           | \( \text{as in Way 1} \) | \( \text{dependent on the previous M mark} \) | |
|           | \( \text{as in Way 1} \) | \( \text{dependent on the previous M mark} \) | |
|           | \( \frac{dh}{dr} = \frac{8}{3} \text{ (cms}^{-1}) \) | \( \frac{dh}{dr} = \frac{8}{3} \text{ (cms}^{-1}) \) | |

| (b) | \[ \frac{dV}{dt} = 200 \implies V = 200t + c \implies \frac{1}{9} \pi h^3 = 200t + c \] | \[ \frac{dV}{dt} = \frac{1}{3} \pi h^2 \text{ o.e.} \] as in Way 1 | B1 |
|     | \( \left( \frac{1}{3} \pi h^2 \right) \frac{dh}{dr} = 200 \) | \( \text{dependent on the previous M mark} \) | M1 |
|     | \[ h = 15, \frac{dh}{dr} = 200 \times \frac{1}{3} \pi (15)^2 \{ = \frac{200}{75\pi} = \frac{600}{225\pi} \} \] | \( \frac{dh}{dr} = \frac{8}{3} \text{ (cms}^{-1}) \) | A1 cao |
### Question 4 Notes

<p>| | |</p>
<table>
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<tr>
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<tbody>
<tr>
<td>4. (a)</td>
<td><strong>Note</strong> Allow M1 for writing down ( r = h \tan 30 )</td>
</tr>
<tr>
<td></td>
<td><strong>Note</strong> Give M0 A0 for writing down ( r = \frac{h \sqrt{3}}{3} ) or ( r = \frac{h}{\sqrt{3}} ) with no evidence of using trigonometry on ( r ) and ( h ) or Pythagoras on ( r ) and ( h )</td>
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<tr>
<td></td>
<td><strong>Note</strong> Give M0 (unless recovered) for evidence of ( \frac{1}{3} \pi r^2 h = \frac{1}{9} \pi h^3 ) leading to either ( r^2 = \frac{1}{3} h^2 ) or ( r = \frac{h \sqrt{3}}{3} ) or ( r = \frac{h}{\sqrt{3}} )</td>
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<tr>
<td>(b)</td>
<td><strong>B1</strong> Correct simplified or un-simplified differentiation of ( V ). E.g. ( \frac{1}{3} \pi h^2 ) or ( \frac{3}{9} \pi h^2 )</td>
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<td></td>
<td><strong>Note</strong> ( \frac{dV}{dh} ) does not have to be explicitly stated, but it should be clear that they are differentiating their ( V )</td>
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<td></td>
<td><strong>M1</strong> ( \left( \text{their} \frac{dV}{dh} \right) \times \frac{dh}{dr} = 200 ) or ( 200 \div \left( \text{their} \frac{dV}{dh} \right) )</td>
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<td></td>
<td><strong>dM1</strong> <em>dependent on the previous M mark</em> ( ) Substitutes ( h = 15 ) into an expression which is a result</td>
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<tr>
<td></td>
<td>of either ( 200 \div \left( \text{their} \frac{dV}{dh} \right) ) or ( 200 \times \frac{1}{\left( \text{their} \frac{dr}{dh} \right)} )</td>
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<tr>
<td></td>
<td><strong>A1</strong> ( \frac{8}{3} ) (units are not required)</td>
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<td></td>
<td><strong>Note</strong> Give final A0 for using ( \frac{dV}{dt} = -200 ) to give ( \frac{dh}{dt} = -\frac{8}{3 \pi} ), unless recovered to ( \frac{dh}{dt} = \frac{8}{3 \pi} )</td>
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<tr>
<td>Question Number</td>
<td>Scheme</td>
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<tr>
<td>5.</td>
<td>$x=1+t-5\sin t$, $y=2-4\cos t$, $-\pi \leq t \leq \pi$, $A(k,2)$, $k &gt; 0$, lies on $C$</td>
</tr>
</tbody>
</table>
| (a)             | \{When $y=2,\} \quad 2=2-4\cos t \Rightarrow t=-\frac{\pi}{2}, \frac{\pi}{2}$ \begin{align*}
k \text{ (or } x) &= 1+\frac{\pi}{2}-5\sin \left(\frac{\pi}{2}\right) \quad \text{or } k \text{ (or } x) = 1-\frac{\pi}{2}-5\sin \left(-\frac{\pi}{2}\right) \\
\{When t=-\frac{\pi}{2}, k>0,\} \quad &k = 6-\frac{\pi}{2} \text{ or } 12-\pi \quad \text{or } k \text{ (or } x) = 6-\frac{\pi}{2} \text{ or } 12-\pi \end{align*} \begin{align*}
\text{Sets } y=2 \text{ to find } t \\
\text{and some evidence of using } t \text{ to find } x=...
\end{align*} | M1 | A1 |
| (b)             | \begin{align*}
\frac{dx}{dt} &= 1-5\cos t, \quad \frac{dy}{dt} = 4\sin t \\
dy &= \frac{4\sin \pi}{1-5\cos \pi} \\
\text{at } t = \frac{\pi}{2}, \quad \frac{dy}{dx} &= \frac{4\sin \left(-\frac{\pi}{2}\right)}{1-5\cos \left(-\frac{\pi}{2}\right)} \\
\text{= -4}
\end{align*} \begin{align*}
\text{At least one of } \frac{dx}{dt} \text{ or } \frac{dy}{dt} \text{ correct (Can be implied)} \\
\text{Both } \frac{dx}{dt} \text{ and } \frac{dy}{dt} \text{ are correct (Can be implied)} \\
\text{Applies their } \frac{dy}{dt} \text{ divided by their } \frac{dx}{dt} \text{ and }
\text{substitutes their } t \text{ into their } \frac{dy}{dx} \\
\text{for this mark}
\end{align*} | B1 | B1 |
|                 | $y-2=-4\left(x-\frac{6-\pi}{2}\right)$ | | M1 |
|                 | $2=\left(-4\right)\left(6-\frac{\pi}{2}\right)+c \Rightarrow y=-4x+2+\left(6-\frac{\pi}{2}\right)$ | | M1 |
|                 | $\{y-2=-4x+24-2\pi \Rightarrow \} \quad y=-4x+26-2\pi$ | | A1 eso |

**Question 5 Notes**

5. (a)  
**Note** M1 can be implied by either $x$ or $k = 6-\frac{\pi}{2}$ or awrt 4.43 or $x$ or $k = \frac{\pi}{2}-4$ or awrt $-2.43$

**Note** An answer of 4.429… without reference to a correct exact answer is A0

**Note** M1 can be earned in part (a) by working in degrees

**Note** Give M0 for not substituting their $t$ back into $x$. E.g. $2=2-4\cos t \Rightarrow t=-\frac{\pi}{2} \Rightarrow k=-\frac{\pi}{2}$

**Note** If two values for $k$ are found, they must identify the correct answer for A1

**Note** Condone M1 for $2=2-4\cos t \Rightarrow t=-\frac{\pi}{2}, \frac{\pi}{2} \Rightarrow x=1-\frac{\pi}{2}-5\sin \left(\frac{\pi}{2}\right)$

(b)  
**Note** The 1st M mark may be implied by their value for $\frac{dy}{dx}$

\begin{align*}
\frac{dy}{dx} &= \frac{4\sin t}{1-5\cos t}, \text{ followed by an answer of } -4 \text{ (from } t=-\frac{\pi}{2}\text{) or } 4 \text{ (from } t=\frac{\pi}{2}\text{)}
\end{align*}

**Note** Give 1st M0 for applying their $\frac{dx}{dt}$ divided by their $\frac{dy}{dx}$ even if they state $\frac{dy}{dx} = \frac{dy}{dt} + \frac{dx}{dt}$

2nd M1  
- applies $y = -2 \text{ (their } m_t)(x-(their \text{ } k))$,
- applies $2 \text{ (their } m_t)(\text{their } k) + c \text{ leading to } y = (\text{their } m_t)x + (\text{their } c)$

where $k$ must be in terms of $\pi$ and $m_t \neq m_x$ is a numerical value found using calculus

**Note** Correct bracketing must be used for 2nd M1, but this mark can be implied by later working
<table>
<thead>
<tr>
<th>Question 5 Notes Continued</th>
</tr>
</thead>
</table>
| **5. (b)** Note | The final A mark is dependent on all previous marks in part (b) being scored.  
This is because the correct answer can follow from an incorrect $\frac{dy}{dx}$ |
| Note | The first 3 marks can be gained by using degrees in part (b) |
| Note | Condone mixing a correct $t$ with an incorrect $x$ or an incorrect $t$ with a correct $x$ for the M marks |
| Note | Allow final A1 for any answer in the form $y = px + q$  
E.g. Allow final A1 for $y = -4x + 26 - 2\pi$, $y = -4x + 2 + 4\left(6 - \frac{\pi}{2}\right)$ or $y = -4x + \left(\frac{52 - 4\pi}{2}\right)$ |
<p>| Note | Do not apply isw in part (b). So, an incorrect answer following from a correct answer is A0 |
| Note | Do not allow $y = 2(-2x + 13 - \pi)$ for A1 |
| Note | $y = -4x + 26 - 2\pi$ followed by $y = 2(-2x + 13 - \pi)$ is condoned for final A1 |</p>
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Notes</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td>$\frac{dy}{dx} = \frac{y^2}{3\cos^2 2x}$; $-\frac{1}{2} &lt; x &lt; \frac{1}{2}$; $y = 2$ at $x = \frac{\pi}{8}$</td>
<td>Separates variables as shown. Can be implied by a correct attempt at integration. Ignore the integral signs.</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$\int \frac{1}{y^2} dy = \int \frac{1}{3\cos^2 2x} dx$</td>
<td></td>
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<tr>
<td></td>
<td>$\int \frac{1}{y^2} dy = \int \frac{1}{3\sec^2 2x} dx$</td>
<td>$\pm \frac{A}{y^2} \rightarrow \pm \frac{B}{y}$; $A, B \neq 0$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$-\frac{1}{y} = \tan \left(\frac{2x}{2}\right) {+c}$</td>
<td>$\pm \tan 2x$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$-\frac{1}{2} = \tan \left(\frac{2\left(-\frac{\pi}{8}\right)}{2}\right) + c$</td>
<td>Use of $x = -\frac{\pi}{8}$ and $y = 2$ in an integrated equation containing a constant of integration. e.g. $c$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$-\frac{1}{2} = -\frac{1}{6} + c \Rightarrow c = -\frac{1}{3}$</td>
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<tr>
<td></td>
<td>$-\frac{1}{y} = \frac{1}{6} \tan 2x - \frac{1}{3}$</td>
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<tr>
<td></td>
<td>$y = \frac{-1}{\tan 2x - \frac{1}{3}}$ or $y = \frac{6}{2 - \tan 2x}$ or $y = \frac{6\cot 2x}{-1 + 2\cot 2x}$ (\left{-\frac{1}{2} &lt; x &lt; \frac{1}{2}\right})</td>
<td>A1 o.e.</td>
<td></td>
</tr>
</tbody>
</table>

**Question 6 Notes**

6. **B1**  
Separates variables as shown. $\frac{dy}{dx}$ and $dx$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. The number “3” may appear on either side.  
E.g. $\int \frac{1}{y^2} dy = \int \frac{1}{3 \sec^2 2x} dx$ or $\int \frac{3}{y^2} dy = \int \frac{1}{\cos^2 2x} dx$ are fine for B1

**Note**  
Allow e.g. $\int \frac{1}{y^2} dy = \int \frac{1}{3 \sec^2 2x} dx$ for B1 or condone $\int \frac{1}{y^2} = \int \frac{1}{3 \sec^2 2x} dx$ for B1

**Note**  
B1 can be implied by correct integration of both sides

**M1**  
$\pm \frac{A}{y^2} \rightarrow \pm \frac{B}{y}$; $A, B \neq 0$

**M1**  
$\frac{1}{\cos^2 2x}$ or $\sec^2 2x \rightarrow \pm \lambda \tan 2x$; $\lambda \neq 0$

**A1**  
$-\frac{1}{y} = \frac{1}{3} \left(\tan \frac{2x}{2}\right)$ with or without ‘$+c$’. E.g. $-\frac{6}{y} = \tan 2x$

**M1**  
Evidence of using both $x = -\frac{\pi}{8}$ and $y = 2$ in an integrated or changed equation containing $c$

**Note**  
This mark can be implied by the correct value of $c$

**Note**  
You may need to use your calculator to check that they have satisfied the final M mark

**Note**  
Condone using $x = \frac{\pi}{8}$ instead of $x = -\frac{\pi}{8}$

**A1**  
$y = \frac{-1}{\tan 2x - \frac{1}{3}}$ or $y = \frac{6}{2 - \tan 2x}$ or any equivalent correct answer in the form $y = f(x)$

**Note**  
You can ignore subsequent working, which follows from a correct answer
### Question 6 Notes Continued

<table>
<thead>
<tr>
<th>6.</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Writing ( \frac{dy}{dx} = \frac{y^2}{3 \cos^2 2x} \Rightarrow \frac{dy}{dx} = \frac{1}{3} y^2 \sec^2 2x ) leading to e.g.</td>
<td></td>
</tr>
<tr>
<td>( y = \frac{1}{9} \sqrt[9]{\tan 2x} ) gets 2(^{nd}) M0 for ( \pm \tan 2x )</td>
<td></td>
</tr>
<tr>
<td>( u = \frac{1}{3} y^2, \frac{dv}{dx} = \sec^2 2x \Rightarrow \frac{du}{dx} = \frac{2}{3} y, \frac{dv}{dx} = \frac{1}{2} \tan 2x ) gets 2(^{nd}) M0 for ( \pm \tan 2x )</td>
<td></td>
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<tr>
<td>because the variables have not been separated</td>
<td></td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
</tr>
<tr>
<td>7.</td>
<td>$\overrightarrow{OA} = \begin{pmatrix} -3 \ 7 \ 2 \end{pmatrix}, \overrightarrow{AB} = \begin{pmatrix} 4 \ -6 \ 2 \end{pmatrix}, \overrightarrow{OP} = \begin{pmatrix} 9 \ 1 \ 8 \end{pmatrix}; \overrightarrow{OQ} = \begin{pmatrix} 9+4\mu \ 1-6\mu \ 8+2\mu \end{pmatrix}$ or $\overrightarrow{OQ} = \begin{pmatrix} 9+2\mu \ 1-3\mu \ 8+\mu \end{pmatrix}$</td>
</tr>
<tr>
<td>(a)</td>
<td>$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} \Rightarrow \begin{pmatrix} -3 \ 7 + 4 \ 2 \end{pmatrix} = (1, 1, 4)$</td>
</tr>
<tr>
<td></td>
<td>$\overrightarrow{OB} = \begin{pmatrix} -3 \ 7 \ 2 \end{pmatrix} + \begin{pmatrix} 4 \ -6 \ 2 \end{pmatrix} = (1, 1, 4)$ or $\begin{pmatrix} 1 \ 1 \ 4 \end{pmatrix}$ or $i + j + 4k$</td>
</tr>
<tr>
<td>Note: M1 can be implied by at least 2 correct components for $B$</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \begin{pmatrix} 9 \ 1 \ 8 \end{pmatrix} - \begin{pmatrix} -3 \ 7 \ 2 \end{pmatrix} = \begin{pmatrix} 12 \ -6 \ 6 \end{pmatrix}$ or $\overrightarrow{PA} = \begin{pmatrix} -12 \ 6 \ -6 \end{pmatrix}$</td>
</tr>
<tr>
<td></td>
<td>$\cos \theta = \frac{\overrightarrow{AP} \cdot \overrightarrow{AB}}{</td>
</tr>
<tr>
<td></td>
<td>$\cos \theta = \frac{96}{216 \cdot \sqrt{56}} \Rightarrow \cos \theta = \frac{4}{\sqrt{21}}$ or $\frac{4}{\sqrt{21}}$</td>
</tr>
<tr>
<td>(c)</td>
<td>$\cos \theta = \frac{4}{\sqrt{21}} \Rightarrow \sin \theta = \sqrt{\frac{21-16}{21}} = \frac{\sqrt{5}}{\sqrt{21}} = \frac{\sqrt{105}}{21}$</td>
</tr>
<tr>
<td></td>
<td>Area $PAB = \frac{1}{2} (\sqrt{216} \cdot \sqrt{56}) = 12 \sqrt{21} \left( \frac{\sqrt{5}}{\sqrt{21}} \right) = 12 \sqrt{5}$</td>
</tr>
<tr>
<td></td>
<td>$12 \sqrt{5}$</td>
</tr>
<tr>
<td>(d)</td>
<td>${ l_2 : } r = \begin{pmatrix} 9 \ 1 + \mu \ 8 \end{pmatrix}$ or $\begin{pmatrix} 9 \ 1 + \mu \ 8 \end{pmatrix}$</td>
</tr>
<tr>
<td></td>
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<tr>
<td>(e)</td>
<td>$\overrightarrow{BQ} = \begin{pmatrix} 9+4\mu \ 1-6\mu \ 8+2\mu \end{pmatrix} - \begin{pmatrix} 1 \ 1 \ 4 \end{pmatrix} = \begin{pmatrix} 8+4\mu \ -6\mu \ 4+2\mu \end{pmatrix}$</td>
</tr>
<tr>
<td></td>
<td>$\overrightarrow{BQ} \cdot \overrightarrow{AP} = 0 \Rightarrow \begin{pmatrix} 8+4\mu \ -6\mu \ 4+2\mu \end{pmatrix} \cdot \begin{pmatrix} 12 \ -6 \ 6 \end{pmatrix} = 0 \Rightarrow \mu = \ldots$</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow 96+48\mu+36\mu+24+12\mu=0 \Rightarrow 96\mu+120=0 \Rightarrow \mu = \frac{5}{4}$</td>
</tr>
<tr>
<td></td>
<td>$\overrightarrow{OQ} = \begin{pmatrix} 9+4(-1.25) \ 1-6(-1.25) \ 8+2(-1.25) \end{pmatrix} = \begin{pmatrix} 4 \ 8.5 \ 5.5 \end{pmatrix} \Rightarrow Q(4, 8.5, 5.5)$</td>
</tr>
<tr>
<td></td>
<td>$\overrightarrow{OQ} = \begin{pmatrix} 9+4(4) \ 1-6(8.5) \ 8+2(5.5) \end{pmatrix} = \begin{pmatrix} 4 \ 8.5 \ 5.5 \end{pmatrix}$ or $\begin{pmatrix} 4 \ 8.5 \ 5.5 \end{pmatrix}$ or $4i + 8.5j + 5.5k$</td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
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</tr>
</tbody>
</table>
7. **Scheme**: 

\[
\overrightarrow{OA} = \begin{pmatrix} -3 \\ 7 \\ 2 \end{pmatrix}, \overrightarrow{AB} = \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}, \overrightarrow{OP} = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix}, \text{ or } \overrightarrow{OQ} = \begin{pmatrix} 9 + 4\mu \\ 1 - 6\mu \\ 8 + 2\mu \end{pmatrix} \quad \text{or} \quad \overrightarrow{OQ} = \begin{pmatrix} 9 + 2\mu \\ 1 - 3\mu \\ 8 + \mu \end{pmatrix}
\]

**Notes**: Let \( \theta \) = size of angle \( \angle PAB \). \( A, B \) lie on \( l_1 \) and \( P \) lies on \( l_2 \).

(e) **Alt 1**

\[
\begin{align*}
\overrightarrow{BQ} &= \begin{pmatrix} 9 + 2\mu \\ 1 - 3\mu \\ 8 + \mu \end{pmatrix} \quad \text{or} \quad \overrightarrow{OB} = \begin{pmatrix} 8 + 2\mu \\ -3\mu \\ -4 - \mu \end{pmatrix} \\
\overrightarrow{BQ} \cdot \overrightarrow{AP} &= 0 \Rightarrow \begin{pmatrix} 8 + 2\mu \\ -3\mu \\ -6 \end{pmatrix} \cdot \begin{pmatrix} 12 \\ -6 \\ 6 \end{pmatrix} = 0 \Rightarrow \mu = ...
\end{align*}
\]

Applies their \( \overrightarrow{OQ} \) – their \( \overrightarrow{OB} \) or their \( \overrightarrow{OB} \) – their \( \overrightarrow{OQ} \)

\[\Rightarrow 96 + 24\mu + 18\mu + 24 + 6\mu = 0 \Rightarrow 48\mu + 120 = 0 \Rightarrow \mu = -\frac{5}{2} \]

\[\mu = -\frac{5}{2} \] A1 o.e.

\[
\overrightarrow{OQ} = \begin{pmatrix} 9 + 2(-2.5) \\ 1 - 3(-2.5) \\ 8 + 1(-2.5) \end{pmatrix} = \begin{pmatrix} 4 \\ 8.5 \\ 5.5 \end{pmatrix} \Rightarrow Q(4, 8.5, 5.5)
\]

Applies vector cross product formula between their \( \overrightarrow{AP} \) or \( \overrightarrow{PA} \) and

\[
\sin \theta = \frac{\sqrt{2880}}{\sqrt{216\times56}} = \frac{\sqrt{5}}{\sqrt{21}} \quad \Rightarrow \cos \theta = \frac{16}{21} = \frac{4}{\sqrt{21}} \quad \text{or} \quad \frac{4\sqrt{21}}{21}
\]

Applies their value of \( \mu \) into \( \overrightarrow{OQ} \) and

\[
\begin{align*}
\sin \theta &= \frac{\sqrt{2880}}{\sqrt{216\times56}} = \frac{\sqrt{5}}{\sqrt{21}} \\
\Rightarrow \cos \theta &= \frac{16}{21} = \frac{4}{\sqrt{21}} \quad \text{or} \quad \frac{4\sqrt{21}}{21}
\end{align*}
\]

\[
\begin{align*}
\frac{\sqrt{2880}}{\sqrt{216\times56}} &= \frac{\sqrt{5}}{\sqrt{21}} \\
\Rightarrow \cos \theta &= \frac{16}{21} = \frac{4}{\sqrt{21}} \quad \text{or} \quad \frac{4\sqrt{21}}{21}
\end{align*}
\]

(b) **Scheme**: Use this scheme if a vector cross product method is being applied

\[
\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} - \begin{pmatrix} -3 \\ 7 \\ 2 \end{pmatrix} = \begin{pmatrix} 12 \\ -6 \\ 6 \end{pmatrix} \quad \text{or} \quad \overrightarrow{PA} = \begin{pmatrix} -12 \\ 6 \\ -6 \end{pmatrix}
\]

An attempt to find \( \overrightarrow{AP} \) or \( \overrightarrow{PA} \)

\[
\begin{align*}
\overrightarrow{d_1} \times \overrightarrow{d_2} &= \begin{pmatrix} 12 \\ -6 \\ 6 \end{pmatrix} \times \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix} = \begin{pmatrix} i & j & k \end{pmatrix} \begin{pmatrix} 12 \\ -6 \\ 6 \end{pmatrix} = 24i + 0j - 48k
\end{align*}
\]

Applies vector cross product formula 

\[
\sin \theta = \frac{\sqrt{2880}}{\sqrt{216\times56}} = \frac{\sqrt{5}}{\sqrt{21}} \quad \Rightarrow \cos \theta = \frac{16}{21} = \frac{4}{\sqrt{21}} \quad \text{or} \quad \frac{4\sqrt{21}}{21}
\]

(b) **Scheme**: Use this scheme if a vector cross product method is being applied

\[
\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} - \begin{pmatrix} -3 \\ 7 \\ 2 \end{pmatrix} = \begin{pmatrix} 12 \\ -6 \\ 6 \end{pmatrix} \quad \text{or} \quad \overrightarrow{PA} = \begin{pmatrix} -12 \\ 6 \\ -6 \end{pmatrix}
\]

An attempt to find \( \overrightarrow{AP} \) or \( \overrightarrow{PA} \)

\[
\begin{align*}
\overrightarrow{AB} &= \sqrt{216}, \overrightarrow{PA} = \sqrt{56} \quad \text{and} \quad \overrightarrow{PB} = \sqrt{80}
\end{align*}
\]

\[
(\sqrt{80})^2 = (\sqrt{216})^2 + (\sqrt{56})^2 - 2(\sqrt{216})(\sqrt{56})\cos \theta
\]

Applies the cosine rule the correct way round

\[
\cos \theta = \frac{216 + 56 - 80}{2\sqrt{216\times56}} = \frac{192}{2\sqrt{216\times56}}
\]

\[
\Rightarrow \cos \theta = \frac{4}{\sqrt{21}} \quad \text{or} \quad \frac{4\sqrt{21}}{21}
\]

(b) **Scheme**: Use this scheme if a vector cross product method is being applied

\[
\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} - \begin{pmatrix} -3 \\ 7 \\ 2 \end{pmatrix} = \begin{pmatrix} 12 \\ -6 \\ 6 \end{pmatrix} \quad \text{or} \quad \overrightarrow{PA} = \begin{pmatrix} -12 \\ 6 \\ -6 \end{pmatrix}
\]

An attempt to find \( \overrightarrow{AP} \) or \( \overrightarrow{PA} \)

\[
\begin{align*}
\overrightarrow{AB} &= \sqrt{216}, \overrightarrow{PA} = \sqrt{56} \quad \text{and} \quad \overrightarrow{PB} = \sqrt{80}
\end{align*}
\]

\[
(\sqrt{80})^2 = (\sqrt{216})^2 + (\sqrt{56})^2 - 2(\sqrt{216})(\sqrt{56})\cos \theta
\]

Applies the cosine rule the correct way round

\[
\cos \theta = \frac{216 + 56 - 80}{2\sqrt{216\times56}} = \frac{192}{2\sqrt{216\times56}}
\]

\[
\Rightarrow \cos \theta = \frac{4}{\sqrt{21}} \quad \text{or} \quad \frac{4\sqrt{21}}{21}
\]

(b) **Scheme**: Use this scheme if a vector cross product method is being applied

\[
\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} - \begin{pmatrix} -3 \\ 7 \\ 2 \end{pmatrix} = \begin{pmatrix} 12 \\ -6 \\ 6 \end{pmatrix} \quad \text{or} \quad \overrightarrow{PA} = \begin{pmatrix} -12 \\ 6 \\ -6 \end{pmatrix}
\]

An attempt to find \( \overrightarrow{AP} \) or \( \overrightarrow{PA} \)

\[
\begin{align*}
\overrightarrow{AB} &= \sqrt{216}, \overrightarrow{PA} = \sqrt{56} \quad \text{and} \quad \overrightarrow{PB} = \sqrt{80}
\end{align*}
\]

\[
(\sqrt{80})^2 = (\sqrt{216})^2 + (\sqrt{56})^2 - 2(\sqrt{216})(\sqrt{56})\cos \theta
\]

Applies the cosine rule the correct way round

\[
\cos \theta = \frac{216 + 56 - 80}{2\sqrt{216\times56}} = \frac{192}{2\sqrt{216\times56}}
\]

\[
\Rightarrow \cos \theta = \frac{4}{\sqrt{21}} \quad \text{or} \quad \frac{4\sqrt{21}}{21}
\]
Question 7 Notes

7. (b) Note If no “subtraction” seen, you can award 1st M1 for 2 out of 3 correct components of the difference
Note For dM1 the dot product formula can be applied as
\[ \sqrt{(12)^2 + (-6)^2 + (6)^2} \cdot \sqrt{(4)^2 + (-6)^2 + (2)^2} \cos \theta = \begin{pmatrix} 12 \\ -6 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix} \]

Note Evaluation of the dot product for \(12i - 6j + 6k\) & \(2i - 3j + k\) is not required for the dM1 mark
A1 For either \(\frac{4}{\sqrt{21}}\) or \(\frac{2}{\sqrt{21}}\) or \(\cos \theta = \frac{4}{\sqrt{21}}\) or \(\frac{2}{\sqrt{21}}\)
Note Using \(12i - 6j + 6k\) & \(2i - 3j + k\) gives \(\cos \theta = \frac{24 + 18 + 6}{12\sqrt{21}} = \frac{48}{12\sqrt{21}} = \frac{4}{\sqrt{21}}\) or \(\frac{2}{\sqrt{21}}\)
Note Using \(2i - j + k\) & \(2i - 3j + k\) gives \(\cos \theta = \frac{4 + 3 + 1}{\sqrt{6}/\sqrt{14}} = \frac{8}{2\sqrt{21}} = \frac{4}{\sqrt{21}}\) or \(\frac{2}{\sqrt{21}}\)
Note Give M1M1A0 for finding \(\theta = \text{awrt} 29.2^\circ\) without reference to \(\cos \theta = \frac{4}{\sqrt{21}}\) or \(\frac{2}{\sqrt{21}}\)
Note Condone taking the dot product between vectors the wrong way round for the M1 dM1 marks
Note Vectors the wrong way round
  - E.g. taking the dot product between \(\overrightarrow{PA}\) and \(\overrightarrow{AB}\) to give \(\cos \theta = -\frac{4}{\sqrt{21}}\) or \(-\frac{4}{21}\sqrt{21}\)
  - with no other working is final A0
  - E.g. taking the dot product between \(\overrightarrow{PA}\) and \(\overrightarrow{AB}\) to give \(\cos \theta = -\frac{4}{\sqrt{21}}\) or \(-\frac{4}{21}\sqrt{21}\)
    followed by \(\cos \theta = \frac{4}{\sqrt{21}}\) or \(\frac{2}{\sqrt{21}}\) or just simply writing \(\frac{4}{\sqrt{21}}\) or \(\frac{2}{\sqrt{21}}\) is final A1
Note In part (b), give M0dM0 for finding and using \(\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{AB} = (5i + 7j + 6k)\)

(c) Note Give 1st M0 for \(\sin \theta = \sin \left(\cos^{-1} \left(\frac{4\sqrt{21}}{21}\right)\right)\) or \(\sin \theta = 1 - \left(\frac{4}{21}\sqrt{21}\right)^2\) unless recovered
M1 Give 2nd M1 for either
  - \(\frac{1}{2}\) (their length \(AP\)) (their length \(AB\)) (their attempt at \(\sin \theta\))
  - \(\frac{1}{2}\) (their length \(AP\)) (their length \(AB\)) sin (their 29.2° from part (b))
  - \(\frac{1}{2}\) (their length \(AP\)) (their length \(AB\)) sin \(\theta\); where \(\cos \theta = \ldots\) in part (b)
Note \(\frac{1}{2}\left(\sqrt{216}\right)(\sqrt{56})\sin(\text{awrt} 29.2^\circ \text{ or awrt} 150.8^\circ) \{= \text{awrt} 26.8\} \text{ without reference to finding } \sin \theta \text{ as an exact value if M0 M1 A0}\)
Note Anything that rounds to 26.8 without reference to finding \(\sin \theta\) as an exact value is M0 M1 A0
Note Anything that rounds to 26.8 without reference to \(12\sqrt{5}\) is A0
Note If they use \(\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{AB} = (5i + 7j + 6k)\) in part (b), then this can be followed through in part (c) for the 2nd M mark as e.g. \(\frac{1}{2}\left(\sqrt{110}\right)(\sqrt{56})\sin \theta\)
Note Finding \(12\sqrt{5}\) in part (c) is M1 dM1 A1, even if there is little or no evidence of finding an exact value for \(\sin \theta\). So \(\frac{1}{2}\left(\sqrt{216}\right)(\sqrt{56})\sin(29.2^\circ) = 12\sqrt{5}\) is M1 dM1 A1
### Question 7 Notes Continued

**7. (d)**

**Note:** Writing \( r = \ldots \) or \( l = \ldots \) or \( Line 2 = \ldots \) is not required for the M mark

A1 Writing \( r = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -6 \\ 2 \end{pmatrix} \) or \( r = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -6 \\ 2 \end{pmatrix} \) or \( r = \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} + \mu d \),

where \( d = \text{a multiple of } 2i - 3j + k \)

**Note:** Writing \( r = \ldots \) or \( l = \ldots \) or \( l = \ldots \) or \( Line 2 = \ldots \) is required for the A mark

**Note:** Other valid \( p = \begin{pmatrix} 1 \\ 8 \end{pmatrix} \) are e.g. \( p = \begin{pmatrix} 13 \\ 6 \\ 10 \end{pmatrix} \) or \( p = \begin{pmatrix} 7 \\ 6 \\ 10 \end{pmatrix} \). So \( r = \begin{pmatrix} 13 \\ -5 \\ 10 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix} \) is M1 A1

**Note:** Give A0 for writing \( l_2 = \ldots \) is required for the A mark

**Note:** Using scalar parameter \( \lambda \) or other scalar parameters (e.g. \( \mu \) or \( s \) or \( t \)) is fine for M1 and/or A1

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<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Notes</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. (c)</td>
<td><strong>Vector Cross Product:</strong> Use this scheme if a vector cross product method is being applied</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \vec{AP} \times \vec{AB} = \begin{pmatrix} 12 \ -6 \ 6 \end{pmatrix} \times \begin{pmatrix} 4 \ -6 \ 2 \end{pmatrix} = 24i + 0j - 48k )</td>
<td>Uses a vector product and ( \sqrt{(24)^2 + (0)^2 + (-48)^2} )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Area ( PAB = \frac{1}{2} \sqrt{(24)^2 + (-48)^2} )</td>
<td>Uses a vector product and ( \frac{1}{2} \sqrt{(24)^2 + (0)^2 + (-48)^2} )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>= 12\sqrt{5} )</td>
<td></td>
<td>A1 cao</td>
</tr>
</tbody>
</table>

| 7. (c) Alt 2    | **Note:** \( \cos \angle PAB = \frac{5}{\sqrt{30}} \) or \( \frac{1}{6} \sqrt{\frac{30}{30}} \) |       |       |
|                | **Note:** \( |\vec{PA}| = \sqrt{216} \) and \( |\vec{PB}| = \sqrt{80} \) |       |       |
|                | \( \sin \theta = \frac{\sqrt{30 - 25}}{\sqrt{30}} = \frac{\sqrt{5}}{\sqrt{30}} = \frac{\sqrt{6}}{6} \) | A correct method for converting an exact value for \( \cos \theta \) to an exact value for \( \sin \theta \) | M1 |
|                | Area \( PAB = \frac{1}{2} \left( \sqrt{216} \right) \left( \sqrt{80} \right) \left( \frac{\sqrt{5}}{\sqrt{30}} \right) = 12\sqrt{30} \left( \frac{\sqrt{5}}{\sqrt{30}} \right) = 12\sqrt{5} \) | \( \frac{1}{2} (\text{their } |\vec{PA}|)(\text{their } |\vec{PB}|) \sin \theta \) | M1 |
|                | = 12\sqrt{5} \) |       | A1 cao |

[3]
### Question 8 Notes

**SC**  **Special Case for the 2nd M and 3rd M mark for those who use their answer from part (a)**

You can apply the 2nd M and 3rd M marks for integration of the form

$$\pm Ax^2 \pm (\text{their answer to part (a)})$$

where their answer to part (a) is in the form

- $$\pm Bx \sin kx \pm C \cos px$$ to give $$\pm Ax^2 \pm Bx \sin kx \pm C \cos px$$
- $$\pm B \sin kx \pm C \sin px$$ to give $$\pm Ax^2 \pm Bx \sin kx \pm C \sin px$$
- $$\pm B \cos kx \pm C \sin px$$ to give $$\pm Ax^2 \pm Bx \cos kx \pm C \sin px$$
- $$\pm B \cos kx \pm C \cos px$$ to give $$\pm Ax^2 \pm Bx \cos kx \pm C \cos px$$

where: $k, p \neq 0$, $k, p$ can be 1
<table>
<thead>
<tr>
<th>Question Number</th>
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<th>Notes</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>8. (b) <strong>Way 2</strong></td>
<td>$V = \pi \int_{0}^{\pi} \left( \sqrt{x} \sin 2x \right)^2 { dx }$</td>
<td>$\pi \int_{0}^{\pi} \left( \sqrt{x} \sin 2x \right)^2 { dx }$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>${ \int x \sin^2 2x , dx = }$</td>
<td>For writing down a correct equation linking $\sin^2 2x$ and $\cos 4x$ (e.g. $\cos 4x = 1 - 2\sin^2 2x$) and some attempt at applying this equation (or a manipulation of this equation which can be incorrect) to their integral. Can be implied</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\int x \left( \frac{1 - \cos 4x}{2} \right) { dx }$</td>
<td>Simplifies $\int x \sin^2 2x { dx }$ to $\int x \left( \frac{1 - \cos 4x}{2} \right) { dx }$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$= x \left( \frac{1}{2} - \frac{1}{8} \sin 4x \right) - \int \left( \frac{1}{2} - \frac{1}{8} \sin 4x \right) { dx }$</td>
<td>Integrates to give $\pm Ax^2 \pm Bx \sin 4x \pm C \cos 4x$; $A, B, C \neq 0$ or an expression that can be simplified to this form</td>
<td>M1 (B1 on ePEN)</td>
</tr>
<tr>
<td></td>
<td>$= x \left( \frac{1}{2} - \frac{1}{8} \sin 4x \right) - \left( \frac{1}{4} x^2 + \frac{1}{32} \cos 4x \right) { + c }$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\left{ \int_{0}^{\pi} \left( \sqrt{x} \sin 2x \right)^2 { dx } = \left[ \frac{1}{4} x^2 - \frac{1}{8} x \sin 4x - \frac{1}{32} \cos 4x \right]_{0}^{\pi} \right}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \left( \frac{1}{4} \left( \frac{\pi}{4} \right)^2 - \frac{1}{8} \sin \left( \frac{4 \pi}{4} \right) - \frac{1}{32} \cos \left( \frac{4 \pi}{4} \right) \right) - \left( 0 - \frac{1}{32} \cos 0 \right)$</td>
<td>dependent on the previous M mark see notes</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>$= \left( \frac{\pi^2}{64} + \frac{1}{16} \right) - \left( - \frac{1}{32} \right) = \frac{\pi^2}{64} + \frac{1}{16}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>So, $V = \pi \left( \frac{\pi^2}{64} + \frac{1}{16} \right)$ or $\frac{1}{64} \pi^3 + \frac{1}{16} \pi$ or $\frac{\pi^2}{2} \left( \frac{\pi^2}{32} + \frac{1}{8} \right)$ o.e.</td>
<td></td>
<td>A1 o.e.</td>
</tr>
</tbody>
</table>

**Question 8 Notes Continued**

8. (a) **SC**

Give **Special Case** M1A0A0 for writing down the correct “by parts” formula and using $u = x$, $\frac{dv}{dx} = \cos 4x$, but making only one error in the application of the correct formula

8. (b) **Note**

You can imply B1 for seeing $\pi \int y^2 \{ dx \}$, followed by $y^2 = \left( \sqrt{x} \sin 2x \right)^2$ or $y^2 = x \sin^2 2x$

**Note**

If the form $\cos 4x = \cos^2 2x - \sin^2 2x$ or $\cos 4x = 2 \cos^2 2x - 1$ is used, the 1st M cannot be gained until $\cos^2 2x$ has been replaced by $\cos^2 2x = 1 - \sin^2 2x$ and the result is applied to their integral

**Note**

Mixing $x'$s and e.g. $\theta$’s:

Condone $\cos 4\theta = 1 - 2\sin^2 2\theta$, $\sin^2 2\theta = \frac{1 - \cos 4\theta}{2}$ or $\lambda \sin^2 2\theta = \lambda \left( \frac{1 - \cos 4\theta}{2} \right)$ if recovered in their integration

**Final M1**

Complete method of applying limits of $\frac{\pi}{4}$ and 0 to all terms of an expression of the form $\pm Ax^2 \pm Bx \sin 4x \pm C \cos 4x$; $A, B, C \neq 0$ and subtracting the correct way round.

**Note**

For the final M1 mark in Way 1, allow one transcription error (on $\sin 4x$ or $\cos 4x$) in the copying of their answer from part (a) to part (b)
Evidence of a proper consideration of the limit of 0 on \( \cos 4x \) *where applicable* is needed for the final M mark.

E.g. \[ \left[ \frac{1}{4} x^2 - \frac{1}{8} x \sin 4x - \frac{1}{32} \cos 4x \right] = \]

- \[ \left( \frac{\pi^2}{4} \right) - \frac{\pi}{8} \sin \left( \frac{\pi}{4} \right) - \frac{1}{32} \cos \left( \frac{\pi}{4} \right) \] is final M1
- \[ \left( \frac{\pi^2}{4} \right) - \frac{\pi}{8} \sin \left( \frac{\pi}{4} \right) - \frac{1}{32} \cos \left( \frac{\pi}{4} \right) \] - 0 is final M0
- \[ \left( \frac{\pi^2}{4} \right) - \frac{\pi}{8} \sin \left( \frac{\pi}{4} \right) - \frac{1}{32} \cos \left( \frac{\pi}{4} \right) \] - \[ \left( \frac{\pi}{32} \right) \] is final M0 (adding)
- \[ \left( \frac{\pi^2}{4} \right) - \frac{\pi}{8} \sin \left( \frac{\pi}{4} \right) - \frac{1}{32} \cos \left( \frac{\pi}{4} \right) \] - \( (0 + 0 + 0) \) is final M0

**Alternative Method:**

\[
\begin{align*}
&\left\{ \begin{array}{l}
u = \sin^2 2x \\
v = \frac{1}{2} x^2
\end{array} \right\}, \\
&\left\{ \begin{array}{l}
u = x^2 \\
v = -\frac{1}{4} \cos 4x
\end{array} \right\}
\]

\[
\int x \sin^2 2x \, dx
\]

\[
= \frac{1}{2} x^2 \sin^2 2x - \int \frac{1}{2} x^2 (2 \sin 4x) \, dx
\]

\[
= \frac{1}{2} x^2 \sin^2 2x - \int x^3 \sin 4x \, dx
\]

\[
= \frac{1}{2} x^2 \sin^2 2x - \left( -\frac{1}{4} x^3 \cos 4x - \int 2x \left( -\frac{1}{4} \cos 4x \right) \, dx \right)
\]

\[
= \frac{1}{2} x^2 \sin^2 2x - \left( -\frac{1}{4} x^3 \cos 4x + \frac{1}{2} \int x \cos 4x \, dx \right)
\]

\[
= \frac{1}{2} x^2 \sin^2 2x + \frac{1}{4} x^3 \cos 4x - \frac{1}{2} \int x \cos 4x \, dx
\]

\[
= \frac{1}{2} x^2 \sin^2 2x + \frac{1}{4} x^3 \cos 4x - \left( \frac{1}{16} \sin 4x + \frac{1}{16} \cos 4x \right) \{ + c \}
\]

\[
= \frac{1}{2} x^2 \sin^2 2x + \frac{1}{4} x^3 \cos 4x - \frac{1}{8} x \sin 4x - \frac{1}{32} \cos 4x \{ + c \}
\]

\[
V = \pi \int_0^1 \left( \sqrt{x \sin 2x} \right) \, dx = \pi \left( \frac{\pi^2}{64} + \frac{1}{16} \right) \text{ or } \frac{1}{64} \pi^3 + \frac{1}{16} \pi \text{ or } \frac{\pi^2}{2} \left( \frac{\pi^2}{32} + \frac{1}{8} \right) \text{ o.e.}
\]