

Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Mechanics M4
(6680/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.

e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

(i) should have the correct number of terms

(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct

e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.

e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol \surd will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. If a candidate makes more than one attempt at any question:
- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
6. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of $g = 9.8$ should be given to 2 or 3 SF.
- Use of $g = 9.81$ should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

M(A) Taking moments about A.

N2L Newton's Second Law (Equation of Motion)

NEL Newton's Experimental Law (Newton's Law of Impact)

HL Hooke's Law

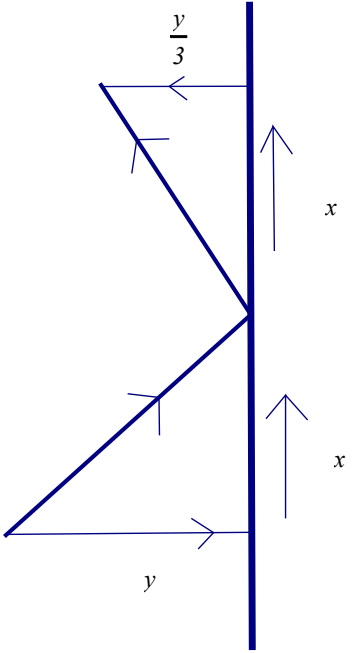
SHM Simple harmonic motion

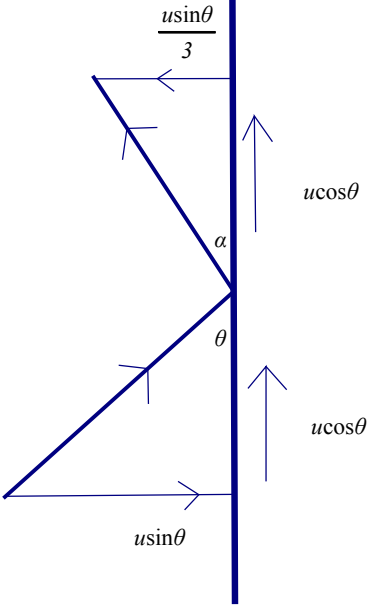
PCLM Principle of conservation of linear momentum

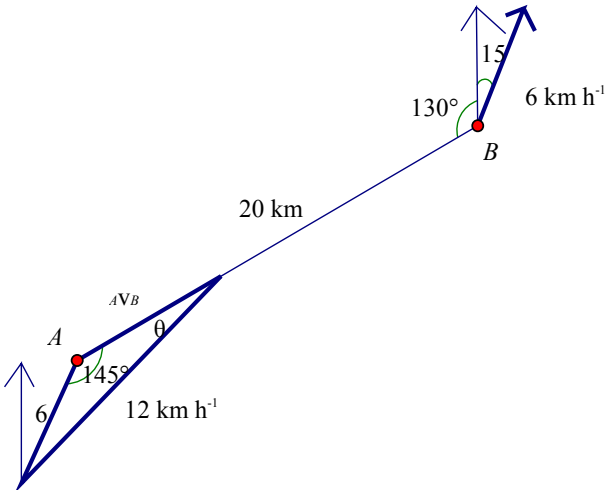
RHS, LHS Right hand side, left hand side.

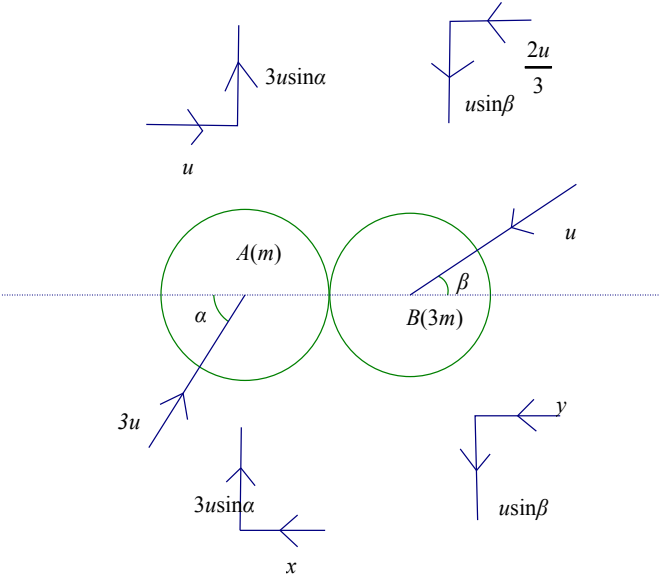
Question Number	Scheme	Marks	Notes
<p>1a</p> <p>alt1</p> <p>alt2</p> <p>alt3</p> <p>1b</p>	$\mathbf{r}_A = (-6\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}) + t(3\mathbf{i} + \mathbf{j}) = ((-6 + 3t)\mathbf{i} + (4 + t)\mathbf{j} + (-3)\mathbf{k})$ $\mathbf{r}_B = (-2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + t(\mathbf{i} - \mathbf{k}) = ((-2 + t)\mathbf{i} + (2)\mathbf{j} + (3 - t)\mathbf{k})$ ${}_B\mathbf{r}_A = (-2 + t + 6 - 3t)\mathbf{i} + (2 - 4 - t)\mathbf{j} + (3 - t + 3)\mathbf{k}$ $= (4 - 2t)\mathbf{i} + (-2 - t)\mathbf{j} + (6 - t)\mathbf{k}$ $ {}_B\mathbf{r}_A ^2 = (4 - 2t)^2 + (t + 2)^2 + (6 - t)^2$ $= 6t^2 - 24t + 56 = 6(t - 2)^2 + 32$ <p>Minimum distance = $\sqrt{32} = 4\sqrt{2}$ m **</p> $ {}_B\mathbf{r}_A ^2 = (4 - 2t)^2 + (t + 2)^2 + (6 - t)^2 (= 6t^2 - 24t + 56)$ $12t - 24 = 0 \Rightarrow t = 2$ <p>Minimum distance = $\sqrt{32} = 4\sqrt{2}$ m **</p> $\begin{pmatrix} 4 - 2t \\ -2 - t \\ 6 - t \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 0 \Rightarrow 8 - 4t - 2 - t + 6 - t = 12 - 6t = 0$ $\text{Distance} = \sqrt{0^2 + 4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$ <p>When $t = 2$,</p> $\mathbf{r}_A = 6\mathbf{j} - 3\mathbf{k}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[6]</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>[2]</p>	<p>Position vector for A or B</p> <p>Both position vectors correct (seen or implied)</p> <p>Position of B relative to A (or A relative to B)</p> <p>Use of Pythagoras</p> <p>Complete the square</p> <p>Reach given answer correctly</p> <p>Use of Pythagoras</p> <p>Differentiate and solve for t</p> <p>Reach given answer correctly</p> <p>Scalar product of position vector with relative velocity = zero and form equation in t</p> <p>Use of Pythagoras</p> <p>Reach given answer correctly</p> <p>Seen or implied</p> <p>cso</p>

Question Number	Scheme	Marks	Notes
2a	$\frac{P}{v} - 10v = ma; \frac{25000}{v} - 10v = 1000a$ $v = 20, (\text{m s}^{-2}) \quad a = \frac{\frac{25000}{20} - 10 \times 20}{1000} = \frac{25 - 2}{10}$ $= 1.05 (\text{m s}^{-2}) \quad **$	M1	Equation of motion
2b	$v \frac{dv}{dx} = \frac{\frac{25000}{v} - 10v}{1000} = \frac{25000 - 10v^2}{1000v} = \frac{2500 - v^2}{100v}$ $\int \frac{100v^2}{2500 - v^2} dv = \int 1 dx \quad \left(= 100 \int -1 + \frac{2500}{2500 - v^2} dv \right)$	M1	Differential equation in v and x
alt1	$= 100 \int -1 + \frac{25}{50 - v} + \frac{25}{50 + v} dv$ $x(+C) = 100 \left\{ -v + 25 \ln \left \frac{50 + v}{50 - v} \right \right\}$	A1	Obtain given answer correctly
alt2	$x = 100 \left(-20 + 25 \ln \frac{70}{30} \right) - 100 \left(-10 + 25 \ln \frac{60}{40} \right) = 105 (\text{m})$	DM1	Substitute $v = 20$
alt2	$= 100 \left(v - 50 \operatorname{arc} \tanh \left(\frac{v}{50} \right) \right)$ $x(+C) = 100 \left\{ -v + 25 \ln \left \frac{50 + v}{50 - v} \right \right\}$ $x = 100 \left(-20 + 25 \ln \frac{70}{30} \right) - 100 \left(-10 + 25 \ln \frac{60}{40} \right) = 105 (\text{m})$	A1	Separate the variables
	NB A correct numerical answer that does not follow from integration scores no marks.	A1	Split using partial fractions Or equivalent
		A1	Integration correct
		DM1	Correct use of limits
		A1	Or better $\left(2500 \ln \left(\frac{14}{9} \right) - 1000 \right)$
		DM1	Use of arctanh
		A1	correct
		A1	Convert to log form
		DM1	Correct use of limits
		A1	Or better $\left(2500 \ln \left(\frac{14}{9} \right) - 1000 \right)$

Question Number	Scheme	Marks	Notes
<p>3 alt1</p>	 <p>Speed perpendicular to wall after collision = $\frac{y}{3}$</p> <p>Speed parallel to the wall is unchanged</p> $\frac{1}{2}(x^2 + y^2) = x^2 + \frac{1}{9}y^2$ $9(x^2 + y^2) = 2(9x^2 + y^2), \quad 9x^2 = 7y^2, \quad x = \frac{\sqrt{7}}{3}y$ <p>direction deflected by $\tan^{-1} \frac{y}{x} + \tan^{-1} \frac{y}{3x}$</p> $= \tan^{-1} \sqrt{\frac{27}{5}} + \tan^{-1} \sqrt{\frac{3}{5}} = 104.5^\circ \quad (104)$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[8]</p>	<p>Use the speeds to form an equation in x & y (or equivalent)</p> <p>Correct unsimplified</p> <p>Correct ratio for x & y (any equivalent form)</p> <p>To find the correct angle</p> <p>Correct in x & y</p>

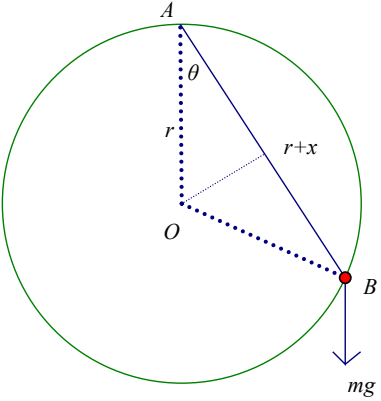
Question Number	Scheme	Marks	Notes
alt2	 <p data-bbox="403 837 1033 902">Speed perpendicular to wall after collision = $\frac{u \sin \theta}{3}$</p> <p data-bbox="403 912 873 945">Speed parallel to the wall is unchanged</p> $\frac{u^2}{4} = \frac{u^2}{9} \sin^2 \theta + u^2 \cos^2 \theta$ $27 \cos^2 \theta = 5 \sin^2 \theta, \quad \tan^2 \theta = \frac{27}{5}$ <p data-bbox="403 1146 1117 1286">deflected by $\theta + \alpha$, $\tan(\theta + \alpha) = \frac{\tan \theta + \frac{1}{3} \tan \theta}{1 - \frac{1}{3} \tan^2 \theta} (= -\sqrt{15})$</p> $\theta + \alpha = 104.5^\circ \quad (104)$	<p data-bbox="1180 857 1222 889">B1</p> <p data-bbox="1180 912 1222 945">B1</p> <p data-bbox="1180 967 1222 1000">M1</p> <p data-bbox="1180 1023 1222 1055">A1</p> <p data-bbox="1180 1078 1222 1110">A1</p> <p data-bbox="1180 1166 1222 1198">M1</p> <p data-bbox="1180 1221 1222 1253">A1</p> <p data-bbox="1180 1276 1222 1308">A1</p> <p data-bbox="1226 1325 1268 1357">[8]</p>	<p data-bbox="1285 948 1818 1013">Use the speeds to form an equation in u & θ (or equivalent)</p> <p data-bbox="1285 1019 1537 1052">Correct unsimplified</p> <p data-bbox="1285 1058 1747 1091">Correct trig ratio for θ (or equivalent)</p> <p data-bbox="1285 1146 1587 1179">To find the correct angle</p> <p data-bbox="1285 1227 1629 1260">Correct in θ (or equivalent)</p>

Question Number	Scheme	Marks	Notes
<p>4a</p>	 <p>Relative velocity triangle</p> $\frac{\sin 145}{12} = \frac{\sin \theta}{6}, \theta = 16.7^\circ$ $\text{Bearing} = 15 + (180 - 145 - 16.7) = 33.3^\circ$ <p>Bearing 033°</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>Seen or implied</p> <p>Use of trig to find a relevant angle</p> <p>To find the required angle</p> <p>They were asked for an answer "to the nearest degree". Accept N 33° E</p>
<p>4b</p>	$\frac{{}_A v_B}{\sin 18.3} = \frac{12}{\sin 145}$ ${}_A v_B = 6.58 (\text{km h}^{-1})$ $\text{Time taken} = \frac{20}{6.58} \text{ (hrs)}$ <p>Time is 3:02 pm (1502)</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>Correct method to find the relative velocity</p> <p>For their 6.58</p>

Question Number	Scheme	Marks	Notes
5a	<p data-bbox="394 310 457 334"><i>Before</i></p>  <p data-bbox="422 716 474 740"><i>After</i></p> <p data-bbox="373 850 1140 886">CLM: $mx + 3my = 3m \times u \cos \beta - m \times 3u \cos \alpha = mu \quad (x + 3y = u)$</p> <p data-bbox="373 943 1056 1019">NEL: $x - y = \frac{1}{5}(3u \cos \alpha + u \cos \beta) \left(= \frac{1}{5} \left(u + \frac{2}{3}u \right) = \frac{1}{3}u \right)$</p> <p data-bbox="380 1073 579 1133">$x = \frac{u}{2}$, or $y = \frac{u}{6}$</p> <p data-bbox="373 1179 1052 1252">Magnitude of the impulse on A = $mu - \left(m \times -\frac{u}{2} \right) = \frac{3mu}{2}$</p>	<p data-bbox="1199 841 1241 865">M1</p> <p data-bbox="1199 894 1241 919">A1</p> <p data-bbox="1199 943 1241 967">M1</p> <p data-bbox="1199 1005 1241 1029">A1</p> <p data-bbox="1199 1062 1262 1086">DM1</p> <p data-bbox="1199 1122 1241 1146">A1</p> <p data-bbox="1199 1170 1241 1195">M1</p> <p data-bbox="1199 1211 1241 1235">A1</p> <p data-bbox="1262 1243 1297 1268">[8]</p>	<p data-bbox="1325 821 1850 881">Terms of correct structure but condone sign errors</p> <p data-bbox="1325 927 1818 987">equation of correct structure but condone sign errors</p> <p data-bbox="1325 1049 1818 1109">Dependent on the two previous M marks. Solve for x or y</p> <p data-bbox="1325 1162 1587 1187">Correct for their x or y</p> <p data-bbox="1325 1211 1524 1235">Must be positive</p>

Question Number	Scheme	Marks	Notes
5b	<p>Component of velocity perpendicular to the line of centres before = component after = $3u \sin \alpha = 3u \times \frac{\sqrt{8}}{3} = \sqrt{8}u$</p> <p>KE lost = $\frac{m}{2} \left(9u^2 - \left(8u^2 + \frac{1}{4}u^2 \right) \right) \left[= \frac{3}{8}mu^2 \right]$</p> <p>Fraction lost = $\frac{\frac{3}{8}}{\frac{9}{2}} = \frac{3}{8} \times \frac{2}{9} = \frac{1}{12}$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p>	<p>Change in KE. Does not need to be a fraction at this stage. Does not need to include the (cancelling) component perpendicular to the line of centre. Correct unsimplified</p>

Question Number	Scheme	Marks	Notes
6a	$m\ddot{x} = 4mv - \frac{5ma \times x}{a} \quad v = -\dot{x}$ $\ddot{x} + 4\dot{x} + 5x = 0 \quad **$	M1 M1 A1 [3]	Equation of motion as far as $m\ddot{x} = \pm 4mv - T$ Use of $v = -\dot{x}$ Reach given answer correctly.
6b	$\text{AE } m^2 + 4m + 5 = 0, \quad m = \frac{-4 \pm \sqrt{4^2 - 4 \times 5}}{2} = -2 \pm i$ $x = e^{-2t} (A \cos t + B \sin t)$ $t = 0, x = a = A$ $\dot{x} = -2e^{-2t} (a \cos t + B \sin t) + e^{-2t} (-a \sin t + B \cos t)$ $t = 0, \quad \dot{x} = 0 = -2a + B \quad x = e^{-2t} (a \cos t + 2a \sin t)$	M1 A1 M1 A1 M1 A1 [6]	Solve AE to find GS Use $t = 0, x = a$ to find A Differentiate and use boundary conditions to find B
6c	$\text{String goes slack when } x = e^{-2t} (a \cos t + 2a \sin t) = 0$ $\cos t = -2 \sin t, \quad \tan t = -\frac{1}{2}$ $\dot{x} = -2e^{-2t} (a \cos t + 2a \sin t) + e^{-2t} (-a \sin t + 2a \cos t)$ $= e^{-2t} (-5a \sin t) = -0.01 \dots a \quad \text{Speed} = 0.011a \text{ (ms}^{-1}\text{)}$	M1 A1 M1 A1 [4]	Set $x = 0$ and solve for t or $\tan t$ Substitute a positive value of t to find the speed. An answer of 0.88... indicates a negative t . The question specifies 2 sf

Question Number	Scheme	Marks	Notes
7a	 <p>Measuring GPE from A, GPE = $-mg \cos \theta (r + x)$</p> $\text{EPE} = \frac{kmgx^2}{2r}$ <p>From the isosceles triangle, $\cos \theta = \frac{x+r}{2r}$</p> $V = -mg \cos \theta (r + x) + \frac{kmgx^2}{2r}$ $= -mg \cos \theta \times 2r \cos \theta + \frac{kmg r^2 (2 \cos \theta - 1)^2}{2r}$ $= mgr \left\{ -2 \cos^2 \theta + 2k \cos^2 \theta - 2k \cos \theta + \frac{k}{2} \right\}$ $= 2mgr \left\{ (k-1) \cos^2 \theta - k \cos \theta \right\} + \text{constant} \quad **$	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(6)</p>	<p>Or $-2mgr \cos^2 \theta$, or $-mgr(1 + \cos 2\theta)$ or equivalent</p> <p>Correct unsimplified total</p> <p>In terms of r & θ</p> <p>Reach given answer correctly</p>

Question Number	Scheme	Marks	Notes
7b	$V = 2mgr(2\cos^2\theta - 3\cos\theta) + \text{constant}$ $V' = 2mgr(-4\cos\theta\sin\theta + 3\sin\theta)$ $V' = 0 \Rightarrow \sin\theta = 0 \text{ or } \cos\theta = \frac{3}{4}$ $\theta = 0 \text{ or } \theta = \pm 0.72 \text{ rads}$ $V'' = 2mgr(-4\cos 2\theta + 3\cos\theta)$ $\theta = 0, V'' = -2mgr < 0, \text{ unstable equilibrium}$ $\cos\theta = \frac{3}{4}, V'' = \frac{7mgr}{2} > 0, \text{ stable equilibrium}$	M1 A1 M1 A3 M1 A1 A1 (9)	Differentiate V Derivative = 0 and solve for θ -1 for each missing solution Second derivative of V Need to see $-2mgr$ or equivalent Do not need to consider the symmetrical position as well

