OXFORD CAMBRIDGE AND RSA EXAMINATIONS
A2 GCE

4723/01

MATHEMATICS

Core Mathematics 3

QUESTION PAPER

MONDAY 16 JUNE 2014: Morning
DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4723/01
List of Formulae (MF1)
Insert for question 6(i)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (RW/SW) 4044131574 90474/1

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Answer ALL the questions.

Read each question carefully. Make sure you know what you have to do before starting your answer.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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1 Given that $y = 4x^2 \ln x$, find the value of $\frac{d^2y}{dx^2}$ when $x = e^2$. [5]

2 By first using appropriate identities, solve the equation

$5 \cos 2\theta \operatorname{cosec} \theta = 2$

for $0° < \theta < 180°$. [6]

3 (i) Use Simpson's rule with four strips to find an approximation to

$$\int_0^2 e^{\sqrt{x}} dx ,$$

giving your answer correct to 3 significant figures. [4]

(ii) Deduce an approximation to $\int_0^2 \left(1 + 10e^{\sqrt{x}}\right) dx$. [2]

4 The functions f and g are defined for all real values of x by

$f(x) = 2x^3 + 4$ and $g(x) = \sqrt[3]{x - 10}$.

(i) Evaluate $f^{-1}(-50)$. [2]

(ii) Show that $fg(x) = 2x - 16$. [2]

(iii) Differentiate $gf(x)$ with respect to x. [3]

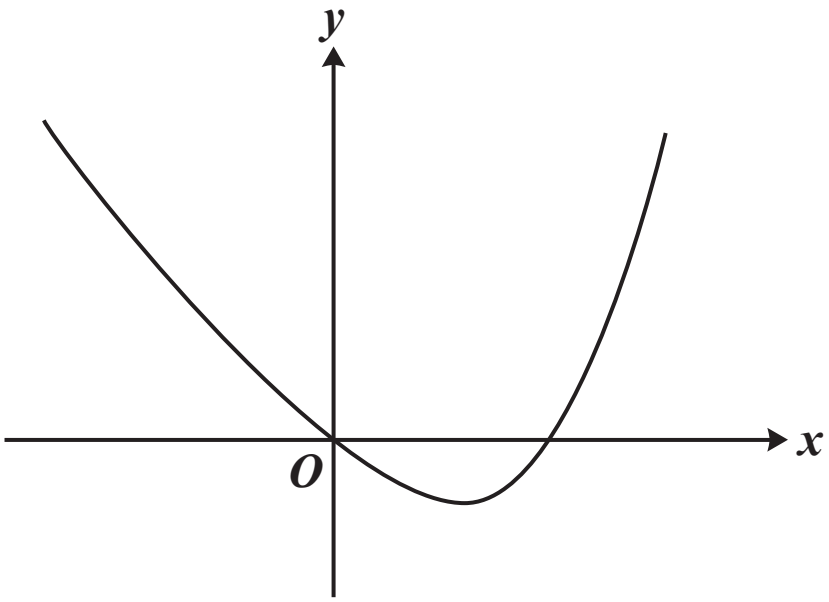
5 (a) The mass, M grams, of a substance at time t years is given by

$$M = 58\mathrm{e}^{-0.33t}.$$

Find the rate at which the mass is decreasing at the instant when $t = 4$. Give your answer correct to 2 significant figures. [3]

(b) The mass of a second substance is increasing exponentially. The initial mass is 42.0 grams and, 6 years later, the mass is 51.8 grams. Find the mass at a time 24 years after the initial value. [4]

6 Look at the following diagram.



The diagram above shows the curve $y = x^4 - 8x$.

(i) By sketching a second curve on the copy of the diagram, show that the equation

$$x^4 + x^2 - 8x - 9 = 0$$

has two real roots. State the equation of the second curve. [2]

(ii) The larger root of the equation $x^4 + x^2 - 8x - 9 = 0$ is denoted by α.

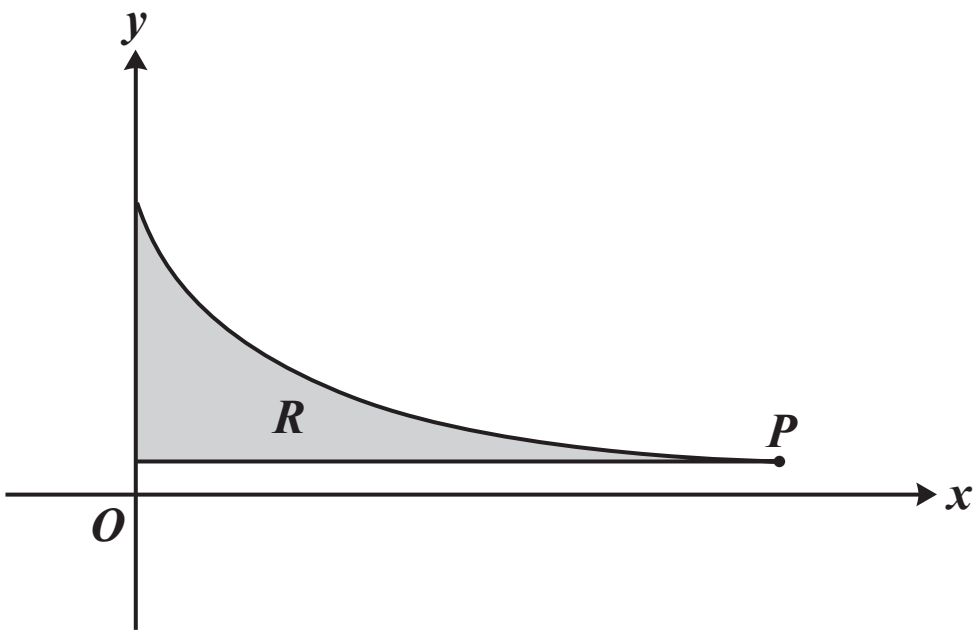
(a) Show by calculation that $2.1 < \alpha < 2.2$. [2]

(b) Use an iterative process based on the equation

$$x = \sqrt[4]{9 + 8x - x^2},$$

with a suitable starting value, to find α correct to 3 decimal places. Give the result of each step of the iterative process. [4]

7 Look at the following diagram.



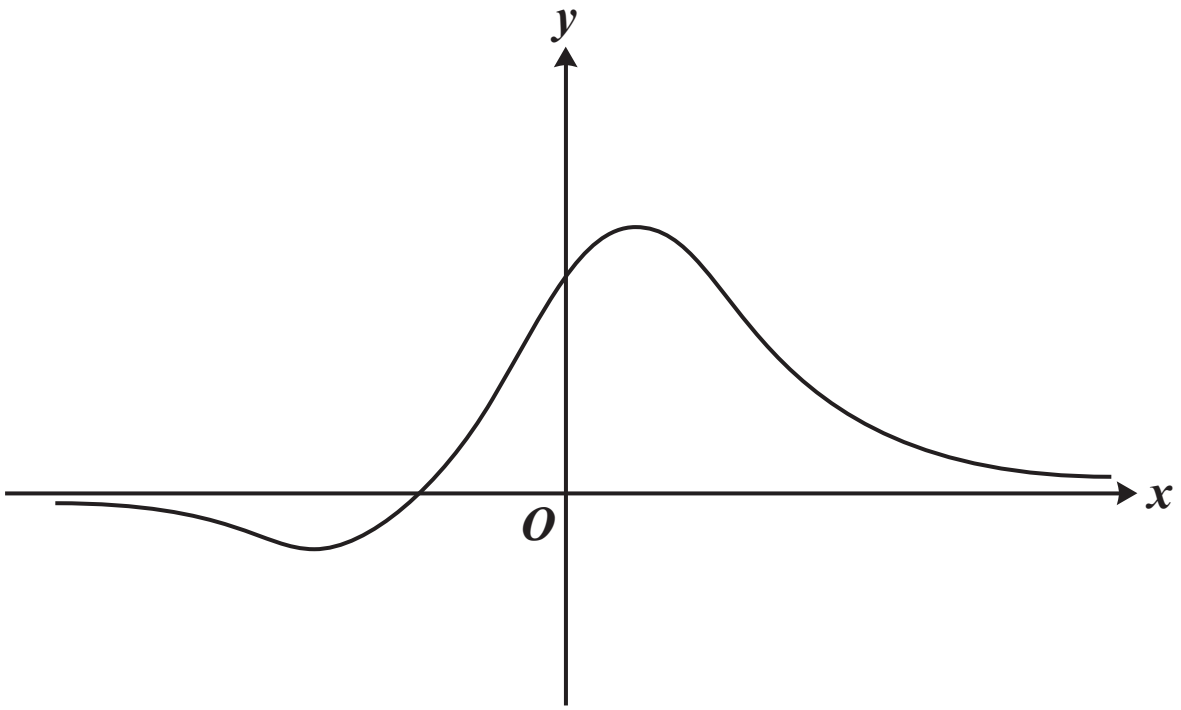
The diagram shows the curve $y = \sqrt{\frac{3}{4x+1}}$ for $0 \leqslant x \leqslant 20$.

The point P on the curve has coordinates $\left(20, \frac{1}{9}\sqrt{3}\right)$.
The shaded region R is enclosed by the curve and the lines $x = 0$ and $y = \frac{1}{9}\sqrt{3}$.

(i) Find the exact area of R. [4]

(ii) Find the exact volume of the solid obtained when R is rotated completely about the x-axis. [6]

8 Look at the following diagram.



The diagram shows the curve $y = \dfrac{2x+4}{x^2+5}$.

(i) Find $\dfrac{dy}{dx}$ and hence find the coordinates of the two stationary points. [6]

(ii) The function g is defined for all real values of x by

$$g(x) = \left|\frac{2x+4}{x^2+5}\right|.$$

(a) Sketch the curve $y = g(x)$ and state the range of g. [3]

(b) It is given that the equation $g(x) = k$, where k is a constant, has exactly two distinct real roots. Write down the set of possible values of k. [2]

9 (i) Express $5\cos(\theta - 60°) + 3\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(ii) Hence

(a) give details of the transformations needed to transform the curve $y = 5\cos(\theta - 60°) + 3\cos\theta$ to the curve $y = \sin\theta$, [3]

(b) find the smallest positive value of β satisfying the equation

$$5\cos\left(\frac{1}{3}\beta - 40°\right) + 3\cos\left(\frac{1}{3}\beta + 20°\right) = 3. \quad [5]$$

END OF QUESTION PAPER

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