OXFORD CAMBRIDGE AND RSA EXAMINATIONS
AS GCE

4725/01

MATHEMATICS

Further Pure Mathematics 1

QUESTION PAPER

FRIDAY 16 MAY 2014: Afternoon
DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:
Printed Answer Book 4725/01
List of Formulae (MF1)

OTHER MATERIALS REQUIRED:
Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (KN) 4032097997 97022

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

If you use the Printed Answer Book, write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

The total number of marks for this paper is 72.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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1 Find the determinant of the matrix $\begin{pmatrix} a & 4 & -1 \\ 3 & a & 2 \\ a & 1 & 1 \end{pmatrix}$. [3]

2 The complex number $7 + 3\text{i}$ is denoted by z. Find

(i) $|z|$ and $\arg z$, [2]

(ii) $\dfrac{z}{4 - \text{i}}$, showing clearly how you obtain your answer. [3]

3 The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -4 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 3 \end{pmatrix}$ and $\mathbf{I}$ is the 2×2 identity matrix. Find

(i) $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$, [2]

(ii) $\mathbf{A}^{-1}$, [2]

(iii) $\left(\mathbf{A}\mathbf{B}^{-1}\right)^{-1}$. [3]

4 (a) Find the matrix that represents a shear with the y-axis invariant, the image of the point (1, 0) being the point (1, 4). [2]

(b) The matrix $\mathbf{X}$ is given by $\mathbf{X} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}$.

(i) Describe fully the geometrical transformation represented by $\mathbf{X}$. [2]

(ii) Find the value of the determinant of $\mathbf{X}$ and describe briefly how this value relates to the transformation represented by $\mathbf{X}$. [2]

5 The cubic equation $2x^3 + 3x + 3 = 0$ has roots α, β and γ.

(i) Use the substitution $x = u + 2$ to find a cubic equation in u. [3]

(ii) Hence find the value of $\frac{1}{\alpha - 2} + \frac{1}{\beta - 2} + \frac{1}{\gamma - 2}$. [4]

6 (i) Show that $\frac{1}{r^2} - \frac{1}{(r+2)^2} \equiv \frac{4(r+1)}{r^2(r+2)^2}$. [2]

(ii) Hence find an expression, in terms of n, for $\sum_{r=1}^{n} \frac{4(r+1)}{r^2(r+2)^2}$. [6]

(iii) Find $\sum_{r=5}^{\infty} \frac{4(r+1)}{r^2(r+2)^2}$, giving your answer in the form $\frac{p}{q}$ where p and q are integers. [2]

7 The loci C_1 and C_2 are given by $\arg(z-2-2\mathrm{i})=\frac{1}{4}\pi$ and $|z|=|z-10|$ respectively.

(i) Sketch on a single Argand diagram the loci C_1 and C_2. [4]

(ii) Indicate, by shading, the region of the Argand diagram for which

$0 \leqslant \arg(z-2-2\mathrm{i}) \leqslant \frac{1}{4}\pi$ and $|z| \geqslant |z-10|$.

[3]

8 (i) Show that $\sum_{r=n}^{2n} r^3 = \frac{3}{4}n^2(n+1)(5n+1)$. [4]

(ii) Hence find $\sum_{r=n}^{2n} r(r^2-2)$, giving your answer in a fully factorised form. [5]

9 The roots of the equation $x^3-kx^2-2=0$ are α, β and γ, where α is real and β and γ are complex.

(i) Show that $k=\alpha-\frac{2}{\alpha^2}$. [2]

(ii) Given that $\beta=u+\mathrm{i}v$, where u and v are real, find u in terms of α. [4]

(iii) Find v^2 in terms of α. [4]

10 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 5^n + 2^{n-1}$.

(i) Find u_1, u_2 and u_3. [2]

(ii) Hence suggest a positive integer, other than 1, which divides exactly into every term of the sequence. [1]

(iii) By considering $u_{n+1} + u_n$, prove by induction that your suggestion in part (ii) is correct. [5]

END OF QUESTION PAPER