OXFORD CAMBRIDGE AND RSA EXAMINATIONS
A2 GCE

MATHEMATICS
4726/01

Further Pure Mathematics 2

QUESTION PAPER

MONDAY 23 JUNE 2014: Morning
DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book, or on any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4726/01
List of Formulae (MF1)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

If you use the Printed Answer Book, write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer <u>ALL</u> the questions.

Do <u>NOT</u> write in the bar codes.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

The total number of marks for this paper is <u>72</u>.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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**1** Find $\displaystyle\int_0^2 \frac{1}{\sqrt{4+x^2}}\,dx$, giving your answer exactly in logarithmic form. [3]

2 It is given that $f(x) = \ln(1+x^2)$.

  (i) Using the standard Maclaurin expansion for $\ln(1+x)$, write down the first four terms in the expansion of $f(x)$, stating the set of values of x for which the expansion is valid. [3]

  (ii) Hence find the exact value of

$$1 - \frac{1}{2}\left(\frac{1}{2}\right)^2 + \frac{1}{3}\left(\frac{1}{2}\right)^4 - \frac{1}{4}\left(\frac{1}{2}\right)^6 + \ldots.$$
[2]

3 The diagram below shows the curve $y = \dfrac{1}{x^3}$ for $1 \leqslant x \leqslant n$ where n is an integer. A set of $(n-1)$ rectangles of unit width is drawn under the curve.
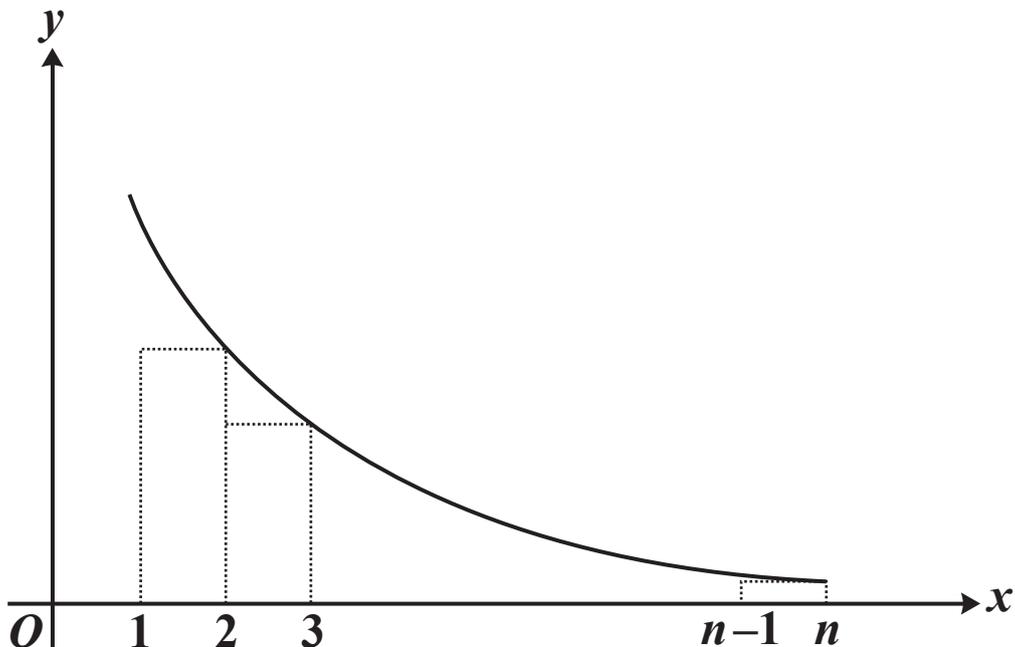
(i) Write down the sum of the areas of the rectangles. [2]

(ii) Hence show that $\displaystyle\sum_{r=1}^{\infty} \frac{1}{r^3} < \frac{3}{2}$. [5]

4 The curves $y = \cos^{-1}x$ and $y = \tan^{-1}\left(\sqrt{2}x\right)$ intersect at a point A.

(i) Verify that the coordinates of A are $\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{4}\pi\right)$. [2]

(ii) Determine whether the tangents to the curves at A are perpendicular. [4]

5

5 A curve has equation $y = \dfrac{x^2 - 8}{x - 3}$.

    (i) Find the equations of the asymptotes of the curve. [3]

    (ii) Prove that there are no points on the curve for which $4 < y < 8$. [4]

    (iii) Sketch the curve. Indicate the asymptotes in your sketch. [2]

6 (i) Given that $y = \cosh^{-1} x$, show that $y = \ln\left(x + \sqrt{x^2 - 1}\right)$. [4]

    (ii) Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\cosh^{-1} x\right) = \dfrac{1}{\sqrt{x^2 - 1}}$. [2]

    (iii) Solve the equation $\cosh x = 3$, giving your answers in logarithmic form. [3]

7 It is given that, for non-negative integers n,

$$I_n = \int_0^{\frac{1}{2}\pi} \sin^n x \, \mathrm{d}x \ .$$

    (i) Show that $I_n = \dfrac{n-1}{n} I_{n-2}$ for $n \geqslant 2$. [3]

    (ii) Explain why $I_{2n+1} < I_{2n-1}$. [2]

    (iii) It is given that $I_{2n+1} < I_{2n} < I_{2n-1}$. Take $n = 5$ to find an interval within which the value of π lies. [6]

8   A curve has polar equation $r = a(1 + \cos\theta)$, where a is a positive constant and $0 \leqslant \theta < 2\pi$.

    (i) Find the equation of the tangent at the pole.   **[2]**

    (ii) Sketch the curve.   **[2]**

    (iii) Find the area enclosed by the curve.   **[6]**

9   The equation $10x - 8\ln x = 28$ has a root α in the interval [3, 4]. The iteration $x_{n+1} = g(x_n)$, where $g(x) = 2.8 + 0.8\ln x$ and $x_1 = 3.8$, is to be used to find α.

    (i) Find the value of α correct to 5 decimal places. You should show the result of each step of the iteration to 6 decimal places.   **[4]**

    (ii) Illustrate this iteration by means of a sketch.   **[2]**

    (iii) The difference, δ_r, between successive approximations is given by $\delta_r = x_{r+1} - x_r$. Find δ_3.   **[2]**

    (iv) Given that $\delta_{n+1} \approx g'(\alpha)\delta_n$, for all positive integers n, estimate the smallest value of n such that $\delta_n < 10^{-6}\delta_1$.   **[4]**

END OF QUESTION PAPER