OXFORD CAMBRIDGE AND RSA EXAMINATIONS
AS GCE
4727/01
MATHEMATICS
Further Pure Mathematics 3
QUESTION PAPER

THURSDAY 12 JUNE 2014: Afternoon
DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:
Printed Answer Book 4727/01
List of Formulae (MF1)

OTHER MATERIALS REQUIRED:
Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (SJF) 4037680980 96921

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

If you use the Printed Answer Book, write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

The total number of marks for this paper is 72.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

1 (i) Find a vector equation of the line of intersection of the planes $2x + y - z = 4$ and $3x + 5y + 2z = 13$. [4]

(ii) Find the exact distance of the point $(2, 5, -2)$ from the plane $2x + y - z = 4$. [2]

2 Use the substitution $u = y^2$ to find the general solution of the differential equation

$$\frac{dy}{dx} - 2y = \frac{e^x}{y}$$

for y in terms of x. [8]

3 (i) Solve the equation $z^6 = 1$, giving your answers in the form $re^{i\theta}$, and sketch an Argand diagram showing the positions of the roots. [4]

(ii) Show that $(1 + i)^6 = -8i$. [3]

(iii) Hence, or otherwise, solve the equation $z^6 + 8i = 0$, giving your answers in the form $re^{i\theta}$. [3]

4 The group G consists of the set $\{1, 3, 7, 9, 11, 13, 17, 19\}$ combined under multiplication modulo 20.

(i) Find the inverse of each element. [3]

(ii) Show that G is not cyclic. [3]

(iii) Find two isomorphic subgroups of order 4 and state an isomorphism between them. [5]

5 Solve the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-x}$$

subject to the conditions $y = \frac{dy}{dx} = 0$ when $x = 0$. [10]

6 The line l has equations $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-7}{5}$.

The plane Π has equation $4x - y - z = 8$.

(i) Show that l is parallel to Π but does not lie in Π. [3]

(ii) The point A $(1, -2, 7)$ is on l. Write down a vector equation of the line through A which is perpendicular to Π. Hence find the position vector of the point on Π which is closest to A. [4]

(iii) Hence write down a vector equation of the line in Π which is parallel to l and closest to it. [1]

7 (i) By expressing $\sin\theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$, show that

$$\sin^5\theta \equiv \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta).$$ [4]

(ii) Hence solve the equation

$$\sin 5\theta + 4\sin\theta = 5\sin 3\theta$$

for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$. [4]

8 G consists of the set of matrices of the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$, where a and b are real and $a^2 + b^2 \neq 0$, combined under the operation of matrix multiplication.

(i) Prove that G is a group. You may assume that matrix multiplication is associative. [6]

(ii) Determine whether G is commutative. [2]

(iii) Find the order of $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. [3]

END OF QUESTION PAPER

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE

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