OXFORD CAMBRIDGE AND RSA EXAMINATIONS

AS GCE

4736/01

MATHEMATICS

Decision Mathematics 1

QUESTION PAPER

TUESDAY 10 JUNE 2014: Morning
DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4736/01
List of Formulae (MF1)
Loose sheet for Question 6

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

The Question Paper will be found inside the Printed Answer Book.

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.

<u>WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK</u>. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Answer <u>ALL</u> the questions.

Read each question carefully. Make sure you know what you have to do before starting your answer.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

The total number of marks for this paper is <u>72</u>.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1   Sangita needs to move some heavy boxes to her new house. She has borrowed a van that can carry at most 600 kg. She will have to make several deliveries to her new house.

The masses of the boxes have been recorded in kg as:

120   120   120   100   150   200   250   150   200

250   120

   (i) Use the first-fit method to show how Sangita could pack the boxes into the van. How many deliveries does this solution require? [3]

   (ii) Use the first-fit decreasing method to show how Sangita could pack the boxes into the van. There is no need to use a sorting algorithm, but you should write down the sorted list before showing the packing. How many deliveries does this solution require? [4]

Sangita then realises that she cannot fit more than four boxes in the van at a time.

   (iii) Find a way to pack the boxes into the van so that she makes as few deliveries as possible. [2]

2 A SIMPLE graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself. A CONNECTED graph is one in which every vertex is joined, directly or indirectly, to every other vertex. A SIMPLY CONNECTED graph is one that is both simple and connected.

(i) (a) Draw a simply connected graph that has exactly four vertices and exactly five arcs. Is your graph Eulerian, semi-Eulerian or neither? Explain how you know. **[3]**

(b) By considering the sum of the vertex orders, show that there is only one possible simply connected graph with exactly four vertices and exactly five arcs. **[5]**

(ii) Draw five distinct simply connected graphs each with exactly five vertices and exactly five arcs. **[3]**

3 The following algorithm finds two positive integers for which the sum of their squares equals a given input, when this is possible.

The function $\text{INT}(X)$ gives the largest integer that is less than or equal to X. For example: $\text{INT}(6.9) = 6$, $\text{INT}(7) = 7$, $\text{INT}(7.1) = 7$.

Line 10	Input a positive integer, N
Line 20	Let $C = 1$
Line 30	If $C^2 \geqslant N$ jump to line 110
Line 40	Let $X = \sqrt{(N - C^2)}$ [you may record your answer as a surd or a decimal]
Line 50	Let $Y = \text{INT}(X)$
Line 60	If $X = Y$ jump to line 100
Line 70	If $C > Y$ jump to line 110
Line 80	Add 1 to C
Line 90	Go back to line 30
Line 100	Print C, X and stop
Line 110	Print 'FAIL' and stop

(i) Apply the algorithm to the input $N = 500$. You only need to write down values when they change and there is no need to record the use of lines 30, 60, 70 or 90. **[4]**

(ii) Apply the algorithm to the input $N = 7$. **[2]**

(iii) Explain why lines 70 and 110 are needed. **[1]**

The algorithm has order $\sqrt{N}$.

(iv) If it takes 0.7 seconds to run the algorithm when $N = 3000$, roughly how long will it take when $N = 12\,000$? **[2]**

4 The network below represents a treasure trail. The arcs represent paths and the weights show distances in units of 100 metres.

The total length of the paths shown is 4200 metres.
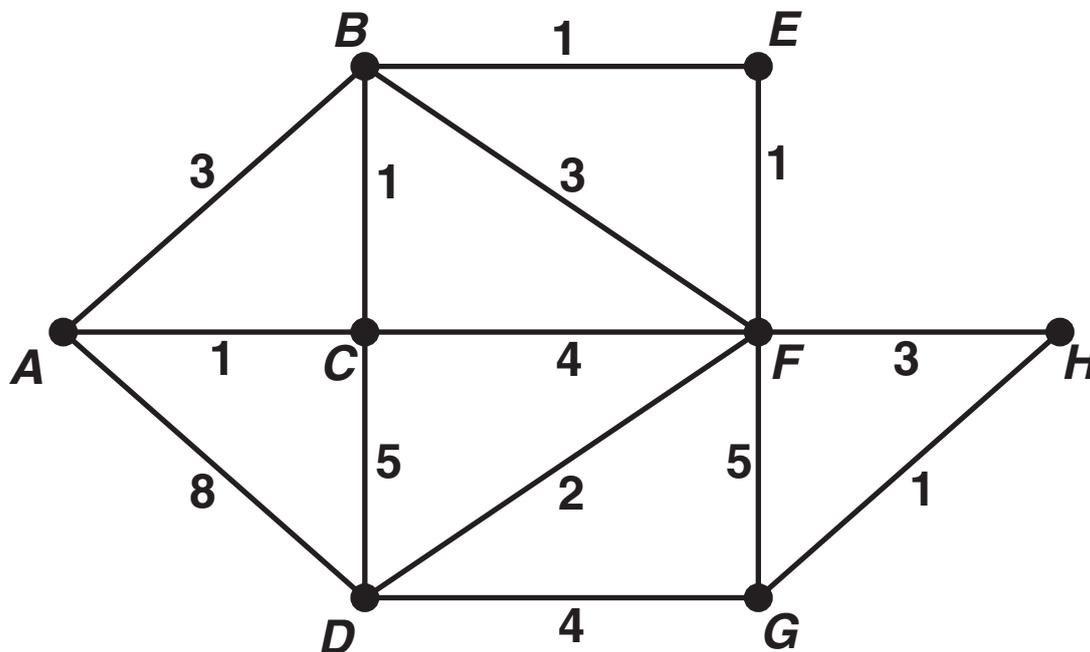
(i) Apply Dijkstra's algorithm to the network, starting at *A*, to find the shortest distance (in metres) from *A* to each of the other vertices. [5]

Alex wants to hunt for the treasure. His current location is marked on the network as *A*. The clues to the location of the treasure are located on the paths. Every path has at least one clue and some paths have more than one. This means that Alex will need to search along the full length of every path to find all the clues.

(ii) Showing your working, find the length of the shortest route that Alex can take, starting and ending at *A*, to find every clue. [3]

The clues tell Alex that the treasure is located at the point marked as *H* on the network.

8

(iii) Write down the shortest route from *A* to *H*. [1]

Zac also starts at *A* and searches along every path to find the clues. He also uses a shortest route to do this, but without returning to *A*. Instead he proceeds directly to the treasure at *H*.

(iv) Calculate the length of the shortest route that Zac can take to search for all the clues and reach the treasure. [2]

5 This question uses the same network as question 4.

The network below represents a treasure trail. The arcs represent paths and the weights show distances in units of 100 metres.
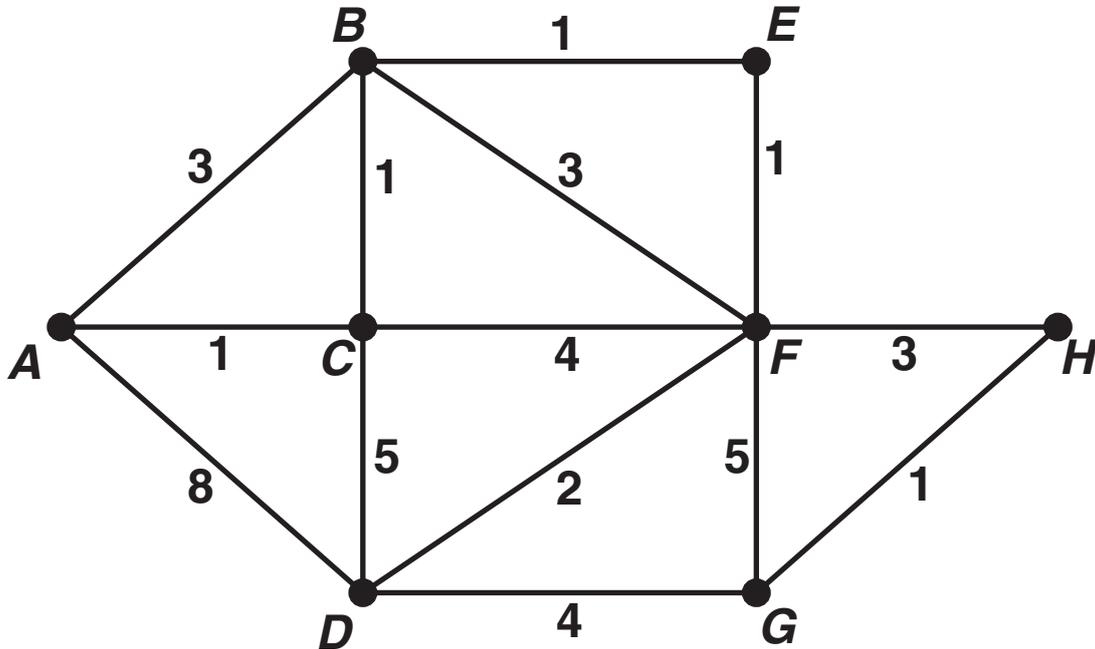
Gus wants to hunt for the treasure. He assumes that the treasure is located at a vertex, but he does not know which one.

  (i) (a) Use the nearest neighbour method starting at *G* to find an upper bound for the length of the shortest closed route through every vertex. [3]

     (b) Gus follows this route, but starting at *A*. Write down his route, starting and ending at *A*. [1]

 (ii) Use Prim's algorithm on the network, starting at *A*, to find a minimum spanning tree. You should write down the arcs in the order they are included, draw the tree and give its total weight (in units of 100 metres). [3]

(iii) (a) Vertex *H* and all arcs joined to *H* are removed from the original network. Write down the weight of the minimum spanning tree for vertices *A*, *B*, *C*, *D*, *E*, *F* and *G* in the resulting reduced network. **[1]**

(b) Use this minimum spanning tree for the reduced network to find a lower bound for the length of the shortest closed route through every vertex in the ORIGINAL network. **[1]**

(iv) Find a route that passes through every vertex, starting and ending at *A*, that is longer than the lower bound from part (iii)(b) but shorter than the upper bound from part (i)(a). Give the length of your route, in metres. **[2]**

Assume that Gus travels along paths at a rate of *x* minutes for every 100 metres and that he spends *y* minutes at each vertex hunting for the treasure. Gus starts by hunting for the treasure at *A*. He then follows the route from part (iv), starting and finishing at *A* and hunting for the treasure at each vertex.

Unknown to Gus, the treasure is found before he gets to it, so he has to search at every vertex. Gus can take at most 2 hours from when he starts searching at *A* to when he arrives back at *A*.

(v) Use this information to write down a constraint on *x* and *y*. **[2]**

The treasure was at *H* and was found 40 minutes after Gus started. This means that Gus takes more than 40 minutes to get to *H*.

(vi) Use this information to write down a second constraint on *x* and *y*. **[2]**

6 Sandie makes tanning lotions which she sells to beauty salons. She makes three different lotions using the same basic ingredients but in different proportions. These lotions are called amber, bronze and copper.

To make one litre of tanning lotion she needs one litre of fluid. This can either be water or water mixed with hempseed oil. One litre of amber lotion uses one litre of water, one litre of bronze lotion uses 0.8 litres of water and one litre of copper lotion uses 0.5 litres of water. Any remainder is made up of hempseed oil. Sandie has 40 litres of water and 7 litres of hempseed oil available.

  (i) By defining appropriate variables a, b and c, show that the constraint on the amount of water available can be written as $10a + 8b + 5c \leqslant 400$. **[2]**

  (ii) Find a similar constraint on the amount of hempseed oil available. **[1]**

The tanning lotions also use two colourants which give two further availability constraints. Sandie wants to maximise her profit, £P. The problem can be represented as a linear programming problem with the initial Simplex tableau on Table 1 on the loose sheet. In this tableau s, t, u and v are slack variables.

  (iii) Use the initial Simplex tableau to write down two inequalities to represent the availability constraints for the colourants. **[2]**

  (iv) Write down the profit that Sandie makes on each litre of amber lotion that she sells. **[1]**

(v) Carry out one iteration of the Simplex algorithm, choosing a pivot from the a column. Show the operations used to calculate each row. [5]

After a second iteration of the Simplex algorithm the tableau is as given on Table 2 on the loose sheet.

(vi) Explain how you know that the optimal solution has been achieved. [1]

(vii) How much of each lotion should Sandie make and what is her maximum profit? Why might the profit be less than this? [4]

(viii) If none of the other availabilities change, what is the least amount of water that Sandie needs to make the amounts of lotion found in part (vii)? [1]

END OF QUESTION PAPER

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