Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Fill in the boxes on the top of the answer book with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D1 answer book provided—there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.
- Do not return the question paper with the answer book.

Information
- The total mark for this paper is 75.
- The marks for each question are shown in brackets—use this as a guide as to how much time to spend on each question.

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
Write your answers in the D1 answer book for this paper.

1. (a) Define the terms

(i) bipartite graph,

(ii) alternating path.

At a hotel, six guests, A, B, C, D, E and F, are to be allocated to six rooms, 1, 2, 3, 4, 5 and 6. Each room needs to be allocated to exactly one guest.

A bipartite graph showing their possible allocations is given in Figure 1. An initial matching is given in Figure 2.

(b) Starting from the initial matching given in Figure 2, apply the maximum matching algorithm to find an alternating path from F to 5. Hence find an improved matching. You must list the alternating path that you use and state your improved matching.

Guest C now has room 5 added to his possible allocations.

(c) Starting with the improved matching found in (b), apply the maximum matching algorithm to obtain a complete matching. You must list the alternating path that you use and state your complete matching.

(Total 10 marks)
Figure 3 represents nine computer terminals, A, B, C, D, E, F, G, H and J, at Pearsonby School. The school wishes to connect them to form a single computer network. The number on each arc represents the cost, in pounds, of connecting the corresponding computer terminals.

(a) Use Prim’s algorithm, starting at B, to find the minimum spanning tree for the computer network. You must clearly state the order in which you select the arcs of your tree.

(b) State the minimum cost of connecting the nine computer terminals.

It is discovered that some computer terminals are already connected. There are already direct connections along BD and FJ, as shown in bold in Diagram 1 in the answer book. It is decided to use these connections.

(c) Use Kruskal’s algorithm to find the minimum spanning tree that includes arcs BD and FJ. You must list the arcs in the order that you consider them. In each case, state whether or not you are adding the arcs to your spanning tree.

(Total 7 marks)
3.  42  21  15  16  35  10  31  11  27  39

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 65

(b) The list of numbers is to be sorted into descending order. Use a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(c) Use the first-fit decreasing bin packing algorithm on your ordered list to pack the numbers into bins of size 65

The nine distinct numbers below are to be sorted into descending order

23  14  17  x  21  18  8  20  11

A bubble sort, starting at the left-hand end of the list, is to be used to obtain the sorted list. After the first complete pass, the list is

23  17  x  21  18  14  20  11  8

After the second complete pass, the list is

23  17  21  18  x  20  14  11  8

(d) Using this information, write down the smallest interval that must contain x. Give your answer as an inequality.

(Total 13 marks)
4.

Figure 4
[The total weight of the network is 85]

Figure 4 represents a network of roads. The number on each edge represents the length, in miles, of the corresponding road. Robyn wishes to travel from A to H. She wishes to minimise the distance she travels.

(a) Use Dijkstra’s algorithm to find the shortest path from A to H. State the shortest path and its length. (6)

On a particular day, Robyn needs to check each road. She must travel along each road at least once. Robyn must start and finish at vertex A.

(b) Use the route inspection algorithm to find the length of the shortest inspection route. State the edges that should be repeated. You should make your method and working clear. (5)

The roads BD and BE become damaged and cannot be used. Robyn needs to travel along all the remaining roads to check that there is no damage to any of them. The inspection route must still start and finish at vertex A.

(c) (i) State the edges that should be repeated.

(ii) State a possible route and calculate its length. You must make your method and working clear. (4)

(Total 15 marks)
Figure 5 shows the constraints of a linear programming problem in $x$ and $y$, where $R$ is the feasible region.

(a) Write down the inequalities that form region $R$.

(b) Find the exact coordinates of the vertices of the feasible region.
The objective is to maximise $P$, where $P = 2x + 3y$

(c) Use point testing to find the optimal vertex, $V$, of the feasible region.

(2)

The objective is changed to maximise $Q$, where $Q = 2x + \lambda y$

Given that $\lambda$ is a constant and $V$ is still the only optimal vertex of the feasible region,

(d) find the range of possible values of $\lambda$.

(4)

(Total 11 marks)
A project is modelled by the activity network shown in Figure 6. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the corresponding activity. Each activity requires exactly one worker. The project is to be completed in the shortest possible time.

(a) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(b) Draw a Gantt chart for the project on the grid provided in the answer book.

(c) State the activities that must be happening at time 18.5

An additional activity, P, is now included in the activity network shown in Figure 6. Activity P is immediately preceded only by activity D. No activity is dependent on the completion of activity P. Each activity still requires exactly one worker and the revised project is to be completed in the shortest possible time.

(d) Explain, briefly, whether or not the revised project can be completed in the same time as the original project if the duration of activity P is

   (i) 10 days

   (ii) 17 days

(Total 11 marks)
7. A caterer can make three different sizes of salad; small, medium and large.

The caterer will make a total of at least 280 salads.

The caterer wants at least 35% of the salads to be small and no more than 20% of the salads to be large.

The caterer has enough ingredients to make 400 small salads or 300 medium salads or 200 large salads.

The profit on each small, medium and large salad is 40p, 60p and 85p respectively. The caterer wants to maximise his total profit.

Let $x$ represent the number of small salads, $y$ represent the number of medium salads and $z$ represent the number of large salads.

Formulate this information as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(Total 8 marks)

TOTAL FOR PAPER: 75 MARKS

END
1. (a)
Question 1 continued

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(Total 10 marks)
Figure 3
Question 2 continued

Diagram 1

(Total 7 marks)
3.  42  21  15  16  35  10  31  11  27  39
Question 3 continued

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(Total 13 marks)
4.

Shortest path: ________________________________________________________

Length of shortest path: ________________________________________________
Question 4 continued

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(Total 15 marks)
Figure 5

$5y + 2x = 50$

$2x + y = 10$

$2y = x$
Question 5 continued
Question 5 continued
Question 5 continued

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(Total 11 marks)
Diagram 1

Key:

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(a)

(b) Leave blank Early event time Late event time

Diagram 1

(c) Leave blank Early event time Late event time

Diagram 1
Question 6 continued
7.
Question 7 continued