Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information
- The total mark for this paper is 75.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
1. A potter believes that 20% of pots break whilst being fired in a kiln. Pots are fired in batches of 25.

(a) Let $X$ denote the number of broken pots in a batch. A batch is selected at random.
Using a 10% significance level, find the critical region for a two tailed test of the potter’s belief. You should state the probability in each tail of your critical region.

The potter aims to reduce the proportion of pots which break in the kiln by increasing the size of the batch fired. He now fires pots in batches of 50. He then chooses a batch at random and discovers there are 6 pots which broke whilst being fired in the kiln.

(b) Test, at the 5% level of significance, whether or not there is evidence that increasing the number of pots in a batch has reduced the percentage of pots that break whilst being fired in the kiln. State your hypotheses clearly.
Question 1 continued
2. A company receives telephone calls at random at a mean rate of 2.5 per hour.

(a) Find the probability that the company receives

(i) at least 4 telephone calls in the next hour,

(ii) exactly 3 telephone calls in the next 15 minutes.

(b) Find, to the nearest minute, the maximum length of time the telephone can be left unattended so that the probability of missing a telephone call is less than 0.2

The company puts an advert in the local newspaper. The number of telephone calls received in a randomly selected 2 hour period after the paper is published is 10

(c) Test at the 5% level of significance whether or not the mean rate of telephone calls has increased. State your hypotheses clearly.
Question 2 continued

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________
Question 2 continued
3. The lifetime, $X$, in tens of hours, of a battery is modelled by the probability density function

$$f(x) = \begin{cases} 
\frac{1}{9} x(4 - x) & 1 \leq x \leq 4 \\
0 & \text{otherwise}
\end{cases}$$

Use algebraic integration to find

(a) $E(X)$

(b) $P(X > 2.5)$

A radio runs using 2 of these batteries, both of which must be working. Two fully-charged batteries are put into the radio.

(c) Find the probability that the radio will be working after 25 hours of use.

(d) Given that the radio is working after 16 hours of use, find the probability that the radio will be working after being used for another 9 hours.
Question 3 continued
4. The continuous random variable \( X \) is uniformly distributed over the interval \( [\alpha, \beta] \)

Given that \( E(X) = 3.5 \) and \( P(X > 5) = \frac{2}{5} \)

(a) find the value of \( \alpha \) and the value of \( \beta \)

(b) (i) find the value of \( c \)

(ii) find \( P(c < X < 9) \)

A rectangle has a perimeter of 200 cm. The length, \( S \) cm, of one side of this rectangle is uniformly distributed between 30 cm and 80 cm.

(c) Find the probability that the length of the shorter side of the rectangle is less than 45 cm.
Question 4 continued
5. The time taken for a randomly selected person to complete a test is \( M \) minutes, where \( M \sim N(14, \sigma^2) \)

Given that 10\% of people take less than 12 minutes to complete the test,

(a) find the value of \( \sigma \) \hspace{1cm} (3)

Graham selects 15 people at random.

(b) Find the probability that fewer than 2 of these people will take less than 12 minutes to complete the test. \hspace{1cm} (3)

Jovanna takes a random sample of \( n \) people.

Using a normal approximation, the probability that fewer than 9 of these \( n \) people will take less than 12 minutes to complete the test is 0.3085 to 4 decimal places.

(c) Find the value of \( n \). \hspace{1cm} (8)
Question 5 continued

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________

Leave blank
Question 5 continued

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________

(Total 14 marks)
6. The continuous random variable $X$ has a probability density function

$$f(x) = \begin{cases} 
  k(x - 2) & 2 \leq x \leq 3 \\
  k & 3 < x < 5 \\
  k(6 - x) & 5 \leq x \leq 6 \\
  0 & \text{otherwise}
\end{cases}$$

where $k$ is a positive constant.

(a) Sketch the graph of $f(x)$. (2)

(b) Show that the value of $k$ is $\frac{1}{3}$ (2)

(c) Define fully the cumulative distribution function $F(x)$. (7)

(d) Hence find the 90th percentile of the distribution. (3)

(e) Find $P[E(X) < X < 5.5]$ (2)
Question 6 continued
Question 6 continued
Question 6 continued

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END