Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information
- The total mark for this paper is 75.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
1. In a call centre, the number of telephone calls, \( X \), received during any 10-minute period follows a Poisson distribution with mean 9.

(a) Find

(i) \( P(X > 5) \)

(ii) \( P(4 \leq X < 10) \)

The length of a working day is 7 hours.

(b) Using a suitable approximation, find the probability that there are fewer than 370 telephone calls in a randomly selected working day.

A week, consisting of 5 working days, is selected at random.

(c) Find the probability that in this week at least 4 working days have fewer than 370 telephone calls.
Question 1 continued

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Question 1 continued

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(Total 12 marks)
2. A fair coin is spun 6 times and the random variable $T$ represents the number of tails obtained.

(a) Give two reasons why a binomial model would be a suitable distribution for modelling $T$.  
(2)

(b) Find $P(T = 5)$  
(2)

(c) Find the probability of obtaining more tails than heads.  
(2)

A second coin is biased such that the probability of obtaining a head is $\frac{1}{4}$.

This second coin is spun 6 times.

(d) Find the probability that, for the second coin, the number of heads obtained is greater than or equal to the number of tails obtained.  
(3)
Question 2 continued
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(Total 9 marks)
3. The length of time, $T$, minutes, spent completing a particular task has probability density function

$$ f(t) = \begin{cases} 
\frac{1}{2}(t - 1) & 1 < t \leq 2 \\
\frac{1}{16}(14t - 3t^2 - 8) & 2 < t \leq 4 \\
0 & \text{otherwise}
\end{cases} $$

(a) Use algebraic integration to find $E(T)$

Given that $E(T^2) = \frac{267}{40}$

(b) find $\text{Var}(T)$

(c) Find the cumulative distribution function $F(t)$

(d) Find the 20th percentile of the time taken to complete the task.

(e) Find the probability that the time spent completing the task is more than 1.5 minutes.

Given that a person has already spent 1.5 minutes on the task,

(f) find the probability that this person takes more than 3 minutes to complete the task.
Question 3 continued
Question 3 continued

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(Total 18 marks)
4. David aims to catch the train to work each morning. The scheduled departure time of the train is 08:30

The number of minutes after 08:30 that the train departs may be modelled by the random variable \( X \). Given that \( X \) has a continuous uniform distribution over \([\alpha, \beta]\) and that \( \text{E}(X) = 4 \) and \( \text{Var}(X) = 12 \)

(a) find the value of \( \alpha \) and the value of \( \beta \). \hspace{1cm} (5)

Each morning, the probability that David oversleeps is 0.05

If David oversleeps he will be late for work.

If he does not oversleep he will be in time to catch the train, but will be late for work if the train departs after 08:35

(b) Find the probability that David will be late for work. \hspace{1cm} (3)

Given that David is late for work,

(c) find the probability that he overslept. \hspace{1cm} (2)
Question 4 continued
Question 4 continued

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5. Past records show that the proportion of customers buying organic vegetables from Tesson supermarket is 0.35

During a particular day, a random sample of 40 customers from Tesson supermarket was taken and 18 of them bought organic vegetables.

(a) Test, at the 5% level of significance, whether or not this provides evidence that the proportion of customers who bought organic vegetables has increased. State your hypotheses clearly. (5)

The manager of Tesson supermarket claims that the proportion of customers buying organic eggs is different from the proportion of those buying organic vegetables. To test this claim the manager decides to take a random sample of 50 customers.

(b) Using a 5% level of significance, find the critical region to enable the Tesson supermarket manager to test her claim. The probability for each tail of the region should be as close as possible to 2.5% (3)

During a particular day, a random sample of 50 customers from Tesson supermarket is taken and 8 of them bought organic eggs.

(c) Using your answer to part (b), state whether or not this sample supports the manager’s claim. Use a 5% level of significance. (1)

(d) State the actual significance level of this test. (1)

The proportion of customers who buy organic fruit from Tesson supermarket is 0.2
During a particular day, a random sample of 200 customers from Tesson supermarket is taken. Using a suitable approximation, the probability that fewer than $n$ of these customers bought organic fruit is 0.0465 correct to 4 decimal places.

(e) Find the value of $n$. (6)
Question 5 continued
Question 5 continued

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(Total 16 marks)
6. The continuous random variable \( X \) has the following cumulative distribution function

\[
F(x) = \begin{cases} 
0 & x \leq 1 \\
\frac{4}{15}(x - 1) & 1 < x \leq 2 \\
k\left(\frac{ax^3}{3} - \frac{x^4}{4}\right) + b & 2 < x \leq 4 \\
1 & x > 4 
\end{cases}
\]

where \( k, a \) and \( b \) are constants.

Given that the mode of \( X \) is \( \frac{8}{3} \),

(a) show that \( a = 4 \) \( \quad (4) \)

(b) Find \( P(X < 2.5) \) giving your answer to 3 significant figures. \( \quad (6) \)
Question 6 continued

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(Total 10 marks)

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TOTAL FOR PAPER: 75 MARKS