Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of $g$ is required, take $g = 9.8 \text{ m s}^{-2}$, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information
- The total mark for this paper is 75.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
A uniform lamina is in the shape of the region $R$. Region $R$ is bounded by the curve with equation $y = 4 - x^2$, the positive $x$-axis and the positive $y$-axis, as shown shaded in Figure 1.

Use algebraic integration to find the $x$ coordinate of the centre of mass of the lamina. (7)
2.

A particle $P$ of mass $m$ is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point $A$. The particle moves in a horizontal circle with constant angular speed $\sqrt{58.8}$ rad s$^{-1}$. The centre $O$ of the circle is vertically below $A$ and the string makes a constant angle $\theta^\circ$ with the downward vertical, as shown in Figure 2.

Given that the tension in the string is $1.2mg$, find

(i) the value of $\theta$

(ii) the length of the string.

(8)
Question 2 continued

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3. A particle $P$ of mass $m$ kg is initially held at rest at the point $O$ on a smooth plane which is inclined at $30^\circ$ to the horizontal. The particle is released from rest and slides down the plane against a force of magnitude $\frac{1}{2}mx^2$ newtons acting towards $O$, where $x$ metres is the distance of $P$ from $O$.

(a) Find the speed of $P$ when $x = 3$ \hspace{1cm} (7) \\

(b) Find the distance $P$ has moved when it first comes to instantaneous rest. \hspace{1cm} (2)
Question 3 continued

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(Total 9 marks)
A thin uniform right hollow cylinder, of radius $a$ and height $4a$, has a base but no top. A thin uniform hemispherical shell, also of radius $a$, is made of the same material as the cylinder. The hemispherical shell is attached to the open end of the cylinder forming a container $C$. The open circular rim of the cylinder coincides with the rim of the hemispherical shell. The centre of the base of $C$ is $O$, as shown in Figure 3.

(a) Find the distance from $O$ to the centre of mass of $C$. 

(b) Find the value of $\theta$. 

The container is placed with its circular base on a plane which is inclined at $\theta^\circ$ to the horizontal. The plane is sufficiently rough to prevent $C$ from sliding. The container is on the point of toppling.
Question 4 continued
Question 4 continued

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Question 4 continued
A hollow cylinder is fixed with its axis horizontal. A particle $P$ moves in a vertical circle, with centre $O$ and radius $a$, on the smooth inner surface of the cylinder. The particle moves in a vertical plane which is perpendicular to the axis of the cylinder. The particle is projected vertically downwards with speed $\sqrt{7ag}$ from the point $A$, where $OA$ is horizontal and $OA = a$. When angle $AOP = \theta$, the speed of $P$ is $v$, as shown in Figure 4.

(a) Show that $v^2 = ag(7 + 2\sin \theta)$  \hspace{1cm} (4)

(b) Verify that $P$ will move in a complete circle. \hspace{1cm} (6)

(c) Find the maximum value of $v$. \hspace{1cm} (2)
Question 5 continued
Question 5 continued

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(Total 12 marks)
6. The ends of a light elastic string, of natural length 0.4 m and modulus of elasticity \( \lambda \) newtons, are attached to two fixed points \( A \) and \( B \) which are 0.6 m apart on a smooth horizontal table. The tension in the string is 8 N.

(a) Show that \( \lambda = 16 \) \( \ldots \) (3)

A particle \( P \) is attached to the midpoint of the string. The particle \( P \) is now pulled \textbf{horizontally} in a direction perpendicular to \( AB \) to a point 0.4 m from the midpoint of \( AB \). The particle is held at rest by a \textbf{horizontal} force of magnitude \( F \) newtons acting in a direction perpendicular to \( AB \), as shown in Figure 5 below.

(b) Find the value of \( F \). \( \ldots \) (4)

The particle is released from rest. Given that the mass of \( P \) is 0.3 kg,

(c) find the speed of \( P \) as it crosses the line \( AB \). \( \ldots \) (6)
Question 6 continued

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Question 6 continued

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Question 6 continued
Question 6 continued
The fixed points $A$ and $B$ are 4 m apart on a smooth horizontal floor. One end of a light elastic string, of natural length 1.8 m and modulus of elasticity 45 N, is attached to a particle $P$ and the other end is attached to $A$. One end of another light elastic string, of natural length 1.2 m and modulus of elasticity 20 N, is attached to $P$ and the other end is attached to $B$. The particle $P$ rests in equilibrium at the point $O$, where $AOB$ is a straight line, as shown in Figure 6.

(a) Show that $AO = 2.2$ m.

The point $C$ lies on the straight line $AOB$ with $AC = 2.7$ m. The mass of $P$ is 0.6 kg. The particle $P$ is held at $C$ and then released from rest.

(b) Show that, while both strings are taut, $P$ moves with simple harmonic motion with centre $O$.

The point $D$ lies on the straight line $AOB$ with $AD = 1.8$ m. When $P$ reaches $D$ the string $PB$ breaks.

(c) Find the time taken by $P$ to move directly from $C$ to $A$. 

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Question 7 continued

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