Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – **there may be more space than you need**.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of \( g \) is required, take \( g = 9.8 \text{ m s}^{-2} \), and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information
- The total mark for this paper is 75.
- The marks for each question are shown in brackets – **use this as a guide as to how much time to spend on each question**.

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
1. [In this question the horizontal unit vectors \( \mathbf{i} \) and \( \mathbf{j} \) are due east and due north respectively.]

A ship \( A \) has constant velocity \((4\mathbf{i} + 2\mathbf{j})\)km\(\text{h}^{-1}\) and a ship \( B \) has constant velocity \((-\mathbf{i} + 3\mathbf{j})\)km\(\text{h}^{-1}\). At noon, the position vectors of the ships \( A \) and \( B \) with respect to a fixed origin \( O \) are \((-2\mathbf{i} + \mathbf{j})\)km and \((5\mathbf{i} - 2\mathbf{j})\)km respectively.

Find

(a) the time at which the two ships are closest together, \( \text{(5)} \)

(b) the length of time for which ship \( A \) is within 2km of ship \( B \). \( \text{(3)} \)
Two smooth uniform spheres $A$ and $B$ have masses $3m$ kg and $m$ kg respectively and equal radii. The spheres are moving on a smooth horizontal surface. Initially, sphere $A$ has velocity $(5i - 2j)\text{ms}^{-1}$ and sphere $B$ has velocity $(3i + 4j)\text{ms}^{-1}$. When the spheres collide, the line joining their centres is parallel to $j$, as shown in Figure 1. The coefficient of restitution between the two spheres is $e$. The kinetic energy of sphere $B$ immediately after the collision is 85% of its kinetic energy immediately before the collision.

Find

(a) the velocity of each sphere immediately after the collision, \hspace{1cm} (9)

(b) the value of $e$. \hspace{1cm} (3)
Question 2 continued

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3. A cyclist and her bicycle have a combined mass of 75 kg. The cyclist travels along a straight horizontal road. The cyclist produces a constant driving force of magnitude 150N. At time \( t \) seconds, the speed of the cyclist is \( v \) m\( \text{s}^{-1} \), where \( v < \sqrt{50} \). As the cyclist moves, the total resistance to motion of the cyclist and her bicycle has magnitude \( 3v^2 \) newtons. The cyclist starts from rest. At time \( t \) seconds, she has travelled a distance \( x \) metres from her starting point.

Find

(a) \( v \) in terms of \( x \),

(b) \( t \) in terms of \( v \).
Question 3 continued
Question 3 continued

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4. [In this question, the unit vectors \( \mathbf{i} \) and \( \mathbf{j} \) are in a vertical plane, \( \mathbf{i} \) being horizontal and \( \mathbf{j} \) being vertically upwards.]

A line of greatest slope of a fixed smooth plane is parallel to the vector \((-4\mathbf{i} - 3\mathbf{j})\). A particle \( P \) falls vertically and strikes the plane. Immediately before the impact, \( P \) has velocity \(-7\mathbf{j}\text{ms}^{-1}\). Immediately after the impact, \( P \) has velocity \((-a\mathbf{i} + \mathbf{j})\text{ms}^{-1}\), where \( a \) is a positive constant.

(a) Show that \( a = 6 \)  

(b) Find the coefficient of restitution between \( P \) and the plane.
Question 4 continued
Question 4 continued

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(Total 8 marks)
5. A cyclist riding due north at a steady speed of 12 km h\(^{-1}\) notices that the wind appears to come from the north-west. At the same time, another cyclist, moving on a bearing of 120° and also riding at a steady speed of 12 km h\(^{-1}\), notices that the wind appears to come from due south. The velocity of the wind is assumed to be constant.

Find

(i) the wind speed,

(ii) the direction from which the wind is blowing, giving your answer as a bearing. (9)
Question 5 continued
Question 5 continued
Question 5 continued

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(Total 9 marks)
6. A particle $P$ of mass 0.2 kg is suspended from a fixed point by a light elastic spring. The spring has natural length 0.8 m and modulus of elasticity 7 N. At time $t = 0$ the particle is released from rest from a point 0.2 metres vertically below its equilibrium position. The motion of $P$ is resisted by a force of magnitude $2v$ newtons, where $v$ m s$^{-1}$ is the speed of $P$. At time $t$ seconds, $P$ is $x$ metres below its equilibrium position.

(a) Show that \[
\frac{d^2x}{dt^2} + 10 \frac{dx}{dt} + 43.75x = 0
\]  
(5)

(b) Find $x$ in terms of $t$.  
(6)

(c) Find the value of $t$ when $P$ first comes to instantaneous rest.  
(2)
Question 6 continued

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(Total 13 marks)
Figure 2 shows four uniform rods, each of mass \( m \) and length \( 2a \). The rods are freely hinged at their ends to form a rhombus \( ABCD \). Point \( A \) is attached to a fixed point on a ceiling and the rhombus hangs freely with \( C \) vertically below \( A \). A light elastic spring of natural length \( 2a \) and modulus of elasticity \( 7mg \) connects the points \( A \) and \( C \). A particle of mass \( 3m \) is attached to point \( C \).

(a) Show that, when \( AD \) is at an angle \( \theta \) to the downward vertical, the potential energy \( V \) of the system is given by

\[
V = 28mga \cos^2 \theta - 48mga \cos \theta + \text{constant}
\]  

Given that \( \theta > 0 \)

(b) find the value of \( \theta \) for which the system is in equilibrium,

(c) determine the stability of this position of equilibrium.
Question 7 continued
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Question 7 continued

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TOTAL FOR PAPER: 75 MARKS

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