Edexcel GCE
Core Mathematics C4
Advanced
Wednesday 18 June 2014 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Total
1. A curve $C$ has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of $x$ and $y$. 

(b) Find an equation of the tangent to $C$ at the point $(3, -2)$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.
2. Given that the binomial expansion of \((1 + kx)^{-4}\), \(|kx| < 1\), is

\[ 1 - 6x + Ax^2 + \ldots \]

(a) find the value of the constant \(k\),

(b) find the value of the constant \(A\), giving your answer in its simplest form.
Question 2 continued
Question 2 continued

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(Total 5 marks)
Figure 1 shows a sketch of part of the curve with equation $y = \frac{10}{2x + 5\sqrt{x}}$, $x > 0$

The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis, and the lines with equations $x = 1$ and $x = 4$

The table below shows corresponding values of $x$ and $y$ for $y = \frac{10}{2x + 5\sqrt{x}}$

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>1.42857</td>
<td>0.90326</td>
<td>0.55556</td>
<td></td>
</tr>
</tbody>
</table>

(a) Complete the table above by giving the missing value of $y$ to 5 decimal places. (1)

(b) Use the trapezium rule, with all the values of $y$ in the completed table, to find an estimate for the area of $R$, giving your answer to 4 decimal places. (3)

(c) By reference to the curve in Figure 1, state, giving a reason, whether your estimate in part (b) is an overestimate or an underestimate for the area of $R$. (1)

(d) Use the substitution $u = \sqrt{x}$, or otherwise, to find the exact value of

$$\int_{1}^{4} \frac{10}{2x + 5\sqrt{x}} \, dx$$

(6)
Question 3 continued
Question 3 continued
A vase with a circular cross-section is shown in Figure 2. Water is flowing into the vase.

When the depth of the water is $h$ cm, the volume of water $V$ cm$^3$ is given by

$$V = 4\pi h(h + 4), \quad 0 \leq h \leq 25$$

Water flows into the vase at a constant rate of $80\pi$ cm$^3$s$^{-1}$

Find the rate of change of the depth of the water, in cm$s^{-1}$, when $h = 6$
Question 4 continued

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(Q4 Total 5 marks)
Figure 3 shows a sketch of the curve $C$ with parametric equations

$$x = 4 \cos \left( t + \frac{\pi}{6} \right), \quad y = 2 \sin t, \quad 0 \leq t < 2\pi$$

(a) Show that

$$x + y = 2\sqrt{3} \cos t$$

(b) Show that a cartesian equation of $C$ is

$$(x + y)^2 + ay^2 = b$$

where $a$ and $b$ are integers to be determined.
6. (i) Find
\[ \int xe^{4x} \, dx \] (3)

(ii) Find
\[ \int \frac{8}{(2x - 1)^3} \, dx, \quad x > \frac{1}{2} \] (2)

(iii) Given that \( y = \frac{\pi}{6} \) at \( x = 0 \), solve the differential equation
\[ \frac{dy}{dx} = e^x \csc 2y \csc y \] (7)
Question 6 continued
Question 6 continued
Question 6 continued

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(Total 12 marks)
7.

Figure 4 shows a sketch of part of the curve \( C \) with parametric equations

\[
x = 3 \tan \theta, \quad y = 4 \cos^2 \theta, \quad 0 \leq \theta < \frac{\pi}{2}
\]

The point \( P \) lies on \( C \) and has coordinates \((3, 2)\).

The line \( l \) is the normal to \( C \) at \( P \). The normal cuts the \( x \)-axis at the point \( Q \).

(a) Find the \( x \)-coordinate of the point \( Q \).

(b) Find the exact value of the volume of the solid of revolution, giving your answer in the form \( p\pi + q\pi^2 \), where \( p \) and \( q \) are rational numbers to be determined.

\[
[You \ may \ use \ the \ formula \ V = \frac{1}{3}\pi r^2 h \ for \ the \ volume \ of \ a \ cone.]
\]
8. Relative to a fixed origin $O$, the point $A$ has position vector \[ \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} \]

and the point $B$ has position vector \[ \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} \]

The line $l_1$ passes through the points $A$ and $B$.

(a) Find the vector $\overrightarrow{AB}$. (2)

(b) Hence find a vector equation for the line $l_1$. (1)

The point $P$ has position vector \[ \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} \]

Given that angle $PBA$ is $\theta$,

(c) show that $\cos \theta = \frac{1}{3}$ (3)

The line $l_2$ passes through the point $P$ and is parallel to the line $l_1$.

(d) Find a vector equation for the line $l_2$ (2)

The points $C$ and $D$ both lie on the line $l_2$.

Given that $AB = PC = DP$ and the $x$ coordinate of $C$ is positive,

(e) find the coordinates of $C$ and the coordinates of $D$. (3)

(f) find the exact area of the trapezium $ABCD$, giving your answer as a simplified surd. (4)
Question 8 continued
Question 8 continued
Question 8 continued
Question 8 continued

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(Total 15 marks)