6686/01

Edexcel GCE

Statistics S4

Advanced/Advanced Subsidiary

Thursday 12 June 2014 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.

Check that you have the correct question paper.

Answer ALL the questions.

You must write your answer to each question in the space following the question.

Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

Full marks may be obtained for answers to ALL questions.

The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).

There are 7 questions in this question paper. The total mark for this paper is 75.

There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You should show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.
1. A production line is designed to fill bottles with oil. The amount of oil placed in a bottle is normally distributed and the mean is set to 100 ml.

The amount of oil, $x$ ml, in each of 8 randomly selected bottles is recorded, and the following statistics are obtained.

\[ \bar{x} = 92.875 \quad s = 8.3055 \]

Malcolm believes that the mean amount of oil placed in a bottle is less than 100 ml.

Stating your hypotheses clearly, test, at the 5% significance level, whether or not Malcolm’s belief is supported.

(5)
Question 1 continued
2. (a) Define

(i) a Type I error,

(ii) a Type II error.

(b) Calculate the probability of a Type I error for this test.

(c) Given that the true mean number of defects per roll of material made by the machine is now 4, calculate the probability of a Type II error.
3. A large number of chicks were fed a special diet for 10 days. A random sample of 9 of these chicks is taken and the weight gained, \( x \) grams, by each chick is recorded. The results are summarised below.

\[
\sum x = 181 \quad \sum x^2 = 3913
\]

You may assume that the weights gained by the chicks are normally distributed.

Calculate a 95% confidence interval for

(a) (i) the mean of the weights gained by the chicks,

(ii) the variance of the weights gained by the chicks.

(10)

A chick which gains less than 16 g has to be given extra feed.

(b) Using appropriate confidence limits from part (a), find the lowest estimate of the proportion of chicks that need extra feed.

(4)
Question 3 continued

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
Question 3 continued

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________

(Total 14 marks)
4. A random sample of 8 people were given a new drug designed to help people sleep.

In a two-week period the drug was given for one week and a placebo (a tablet that contained no drug) was given for one week.

In the first week 4 people, selected at random, were given the drug and the other 4 people were given the placebo. Those who were given the drug in the first week were given the placebo in the second week. Those who were given the placebo in the first week were given the drug in the second week.

The mean numbers of hours of sleep per night for each of the people are shown in the table.

<table>
<thead>
<tr>
<th>Person</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours of sleep with drug</td>
<td>10.8</td>
<td>7.2</td>
<td>8.7</td>
<td>6.8</td>
<td>9.4</td>
<td>10.9</td>
<td>11.1</td>
<td>7.6</td>
</tr>
<tr>
<td>Hours of sleep with placebo</td>
<td>10.0</td>
<td>6.5</td>
<td>9.0</td>
<td>5.6</td>
<td>8.7</td>
<td>8.0</td>
<td>9.8</td>
<td>6.8</td>
</tr>
</tbody>
</table>

(a) State one assumption that needs to be made in order to carry out a paired $t$-test. (1)

(b) Stating your hypotheses clearly, test, at the 1% level of significance, whether or not the drug increases the mean number of hours of sleep per night by more than 10 minutes. State the critical value for this test. (8)
Question 4 continued

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________

Leave blank
Question 4 continued
Question 4 continued
5. A statistician believes a coin is biased and the probability, $p$, of getting a head when the
coin is tossed is less than 0.5

The statistician decides to test this by tossing the coin 10 times and recording the number,
$X$, of heads. He sets up the hypotheses $H_0 : p = 0.5$ and $H_1 : p < 0.5$ and rejects the null
hypothesis if $x < 3$

(a) Find the size of the test. (1)

(b) Show that the power function of this test is

$$(1 - p)^8 (36p^2 + 8p + 1)$$

(3)

Table 1 gives values, to 2 decimal places, of the power function for the statistician’s test.

<table>
<thead>
<tr>
<th>$p$</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
<th>0.35</th>
<th>0.4</th>
<th>0.45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>0.93</td>
<td>0.82</td>
<td>$r$</td>
<td>0.53</td>
<td>0.38</td>
<td>0.26</td>
<td>$s$</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 1

(c) Calculate the value of $r$ and the value of $s$. (2)

**Question 5 parts (d) and (e) continue on page 16**
Question 5 continued

For your convenience Table 1 is repeated here.

<table>
<thead>
<tr>
<th>$p$</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
<th>0.35</th>
<th>0.4</th>
<th>0.45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>0.93</td>
<td>0.82</td>
<td>$r$</td>
<td>0.53</td>
<td>0.38</td>
<td>0.26</td>
<td>$s$</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 1

(d) On the axes below draw the graph of the power function for the statistician’s test.

(2)

(e) Find the range of values of $p$ for which the probability of accepting the coin as unbiased, when in fact it is biased, is less than or equal to 0.4

(3)
Question 5 continued

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________

(Total 11 marks)
6. (a) Explain what is meant by the sampling distribution of an estimator $T$ of the population parameter $\theta$. (1)

(b) Explain what you understand by the statement that $T$ is a biased estimator of $\theta$. (1)

A population has mean $\mu$ and variance $\sigma^2$

A random sample $X_1, X_2, ..., X_{10}$ is taken from this population.

(c) Calculate the bias of each of the following estimators of $\mu$.

\[
\hat{\mu}_1 = \frac{X_1 + X_5 + X_7}{3}, \\
\hat{\mu}_2 = \frac{5X_1 + 2X_2 + X_9}{6}, \\
\hat{\mu}_3 = \frac{3X_{10} - X_1}{3}
\]  (4)

(d) Find the variance of each of these three estimators. (6)

(e) State, giving a reason, which of these three estimators for $\mu$ is

(i) the best estimator,

(ii) the worst estimator. (3)
Question 6 continued
Question 6 continued
Question 6 continued

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

(Total 15 marks)
7. Two groups of students take the same examination.

A random sample of students is taken from each of the groups.

The marks of the 9 students from Group 1 are as follows

\[30 \ 29 \ 35 \ 27 \ 23 \ 33 \ 33 \ 35 \ 28\]

The marks, \(x\), of the 7 students from Group 2 gave the following statistics

\[\bar{x} = 31.29 \quad s^2 = 12.9\]

A test is to be carried out to see whether or not there is a difference between the mean marks of the two groups of students.

You may assume that the samples are taken from normally distributed populations and that they are independent.

(a) State one other assumption that must be made in order to apply this test and show that this assumption is reasonable by testing it at a 10% level of significance. State your hypotheses clearly.

(7)

(b) Stating your hypotheses clearly, test, using a significance level of 5%, whether or not there is a difference between the mean marks of the two groups of students.

(7)
Question 7 continued

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
Question 7 continued

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
(Total 14 marks)