



GCE AS/A level

0975/01

MATHEMATICS – C3
Pure Mathematics

P.M. FRIDAY, 6 June 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_0^3 \ln(8 + e^x) dx.$$

Show your working and give your answer correct to two decimal places. [4]

- (b) Use your answer to part (a) to deduce an approximate value for the integral

$$\int_0^3 \ln(16 + 2e^x) dx. \quad [2]$$

2. Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$8 \tan^2 \theta - 5 \sec^2 \theta = 7 + 4 \sec \theta. \quad [6]$$

3. The curve C is defined by

$$y^4 - 2x^2 + 8xy^2 + 9 = 0.$$

- (a) Show that $\frac{dy}{dx} = \frac{x - 2y^2}{y^3 + 4xy}$. [4]

- (b) Show that there is no point on C at which $\frac{dy}{dx} = 0$. [4]

4. Given that $x = 2e^t - 5$, $y = 8e^{-t} + 3e^t - 4$, find the value of t when $\frac{dy}{dx} = -1$.

Give your answer correct to three decimal places. [7]

5. (a) Show that $f(x) = \ln(3x^2 - 2x - 1) - 4x^2$ has a stationary value when x satisfies

$$12x^3 - 8x^2 - 7x + 1 = 0. \quad [4]$$

- (b) You may assume that the equation $12x^3 - 8x^2 - 7x + 1 = 0$ has a root α between -1 and 0 .

The recurrence relation

$$x_{n+1} = \left(\frac{8x_n^2 + 7x_n - 1}{12} \right)^{\frac{1}{3}}$$

with $x_0 = -0.6$ can be used to find α . Find and record the values of x_1, x_2, x_3, x_4 . Write down the value of x_4 correct to four decimal places and show this is the value of α correct to four decimal places. [5]

6. (a) Differentiate each of the following with respect to x , simplifying your answer wherever possible.

(i) $\frac{1}{\sqrt[4]{9-4x^5}}$ (ii) $\frac{3+2x^3}{7-x^3}$ [5]

- (b) (i) Sketch the graph of $y = \sin^{-1}x$ for values of x satisfying $-1 \leq x \leq 1$.
 (ii) By first rewriting $y = \sin^{-1}x$ as $x = \sin y$, find an expression for $\frac{dy}{dx}$ in terms of x . You should justify any choice of sign that you make. [6]

7. (a) Find each of the following, simplifying your answer wherever possible.

(i) $\int \cos(2-5x) dx$, (ii) $\int \frac{4}{e^{3x-2}} dx$, (iii) $\int \frac{5}{\frac{1}{6}x-3} dx$. [6]

(b) Evaluate $\int_2^6 \sqrt{4x+1} dx$. [4]

8. (a) Show, by counter-example, that the statement

$$|2a + 3b| \equiv 2|a| + 3|b|$$

is false. [2]

- (b) Solve the equation

$$|3x - 2| = 7|x|. \quad [3]$$

9. The function f has domain $(-\infty, 4)$ and is defined by

$$f(x) = x^2 - 8x + 7.$$

- (a) Express $f(x)$ in the form

$$f(x) = (x + a)^2 + b,$$

where a, b are constants whose values are to be found. [1]

- (b) Hence or otherwise, find an expression for $f^{-1}(x)$. [4]

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10. The functions f and g have domains $[-2, \infty)$ and $[2, \infty)$ respectively and are defined by

$$\begin{aligned}f(x) &= x^2 + kx - 8, \\g(x) &= kx - 4,\end{aligned}$$

where k is a positive constant.

- (a) Write down, in terms of k , the range of g . [1]
- (b) (i) Find the least value of k so that the function fg can be formed.
- (ii) Write down an expression in terms of k for $fg(x)$.
- (iii) Given that $fg(3) = 0$, find the value of k . [7]

END OF PAPER