GCE AS/A level

0982/01

MATHEMATICS – M3
Mechanics

A.M. MONDAY, 23 June 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A car of mass 1200 kg is initially at rest on a straight horizontal road. The car moves under the action of a horizontal tractive force of 500 N. The resistance to motion of the car is $100v$ N, where v ms^{-1} is the speed of the car at time t s.

(a) Show that the motion of the car satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{5 - v}{12}.$$ [2]

(b) Find an expression for v in terms of t and write down the limiting speed of the car. [6]

(c) Calculate the time taken for the car to reach a speed of 4 ms^{-1}. [2]

2. A light spring, which is attached at one end to a fixed point, is hanging vertically with a particle attached to the other end. The particle is performing a motion which satisfies the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -k^2x,$$

where x m is the additional extension of the spring from the equilibrium position at time t s, and k is a constant.

(a) Find the value of k for which the period of the motion is 2 s. [2]

(b) Initially, the particle is at rest in the equilibrium position. The particle is then pulled to the position where x = 0·52 and then released.

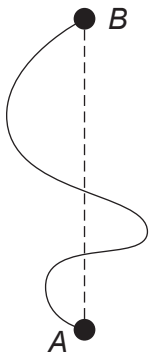
Calculate the value of x when $t = \frac{1}{3}$. [3]

(c) Determine the first two values of t for which x = 0·4. [3]

(d) Calculate the speed of the particle when x = 0·2. [3]

(e) Calculate the maximum speed of the particle. [2]

3. Two particles *A* and *B*, of mass 3 kg and 2 kg respectively, are attached one to each end of a light inextensible string of length $2l$ m. Initially, the particles are at rest on a smooth horizontal surface a distance l m apart, as shown in the diagram. Particle *B* is then projected horizontally with speed 8 ms^{-1} at an angle of 90° to the line joining the initial positions of *A* and *B*.



Find the speed with which each particle begins to move immediately after the string becomes taut and determine the magnitude of the impulsive tension in the string. [9]

4. The reading x of the pointer on a set of kitchen scales at time t is modelled by the differential equation

$$2\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 6\frac{\mathrm{d}x}{\mathrm{d}t} + 5x = 1.$$

(a) Find the general solution of the equation for x. [5]

(b) Determine the limiting value of x. [2]

(c) Given that $x = 0{\cdot}5$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$ when $t = 0$,

(i) find an expression for x in terms of t,

(ii) calculate the instantaneous reading of the scale when $t = \frac{\pi}{3}$.

Give your answer correct to three significant figures. [5]

5. A vehicle of mass 800 kg is being pulled along a straight horizontal road starting from rest at the point *O*, when $t = 0$. At time t s, the vehicle is x m from the point *O* and its velocity is v ms^{-1}. The magnitude of the tractive force can be modelled by $1200(v + 3)^{-1}$ N. Resistance to motion of the vehicle may be ignored.

(a) Show that v satisfies the differential equation

$$2v\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{3}{v+3}.$$ [2]

(b) Show that when the velocity is 3 ms^{-1}, the vehicle has travelled 15 m. [5]

(c) Write down an expression for $\frac{\mathrm{d}v}{\mathrm{d}t}$ in terms of v and hence determine the time taken for the vehicle to reach a velocity of 3 ms^{-1}. [4]

(d) (i) Show that the velocity v of the vehicle at time t s can be given by

$$v = -3 + \sqrt{9 + 3t}.$$

(ii) Verify that the vehicle has travelled approximately 9·5 m after 7 s. [7]

6. A uniform ladder *AB*, of length 8 m and mass 12 kg, rests with its top end *A* against a smooth vertical wall and its bottom end *B* on rough horizontal ground. The ladder is inclined at an angle of 75° to the horizontal. A light inextensible rope is attached to the ladder 3 m from the bottom and tied to the wall so that the rope is horizontal. The rope and the ladder are in the same vertical plane. The coefficient of friction between the ladder and the ground is 0·1. A man of mass 70 kg climbs the ladder.

(a) Draw a diagram clearly showing all the forces acting on the ladder. [2]

(b) Write down the magnitude of the normal reaction of the ground on the ladder. [2]

(c) The rope will break when the tension reaches 100 N. Determine how far up the ladder the man can climb before the rope breaks. [8]

(d) State one modelling assumption you have made about the ladder in your solution. [1]

END OF PAPER