GCE AS/A level

0978/01

S16-0978-01

MATHEMATICS – FP2
Further Pure Mathematics

A.M. FRIDAY, 24 June 2016

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Using the substitution $u = x^2$, evaluate the integral

$$\int_0^{\sqrt{2}} \frac{x}{\sqrt{16-x^4}}\,\mathrm{d}x\,,$$

giving your answer in the form $\dfrac{\pi}{n}$, where n is a positive integer. [6]

2. *(a)* (i) Evaluate $(3-\mathrm{i})^2$, giving your answer in the form $a + \mathrm{i}b$.

(ii) Using your result, show that

$$(3-\mathrm{i})^4 = 28 - 96\mathrm{i}.$$ [3]

(b) Hence write down the four 4th roots of $28 - 96\mathrm{i}$. [3]

3. *(a)* Use de Moivre's Theorem to prove that, for $\sin\theta \neq 0$,

$$\frac{\sin 4\theta}{\sin\theta} = 4\cos\theta\left(1-2\sin^2\theta\right).$$ [4]

(b) Hence evaluate

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sin 4\theta}{\sin\theta}\,\mathrm{d}\theta.$$

Give your answer correct to three significant figures. [4]

4. Using the substitution $t = \tan\left(\dfrac{x}{2}\right)$, find the general solution, in radians, to the equation

$$\sin x + \tan x + \tan\left(\frac{x}{2}\right) = 0\,.$$ [11]

5. The function f is defined by

$$f(x) = \frac{3x^2 + x + 6}{(x+2)\left(x^2+4\right)}\,.$$

(a) Determine whether f is even, odd or neither even nor odd. [1]

(b) Express $f(x)$ in partial fractions. [5]

(c) Hence evaluate

$$\int_0^1 f(x)\,\mathrm{d}x\,,$$

giving your answer correct to three significant figures. [6]

6. *(a)* Show that the general hyperbola with equation

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

can be represented parametrically by $x = a\sec\theta$, $y = b\tan\theta$. [2]

(b) The equation of the hyperbola *H* is

$$x^2 - y^2 = 1.$$

(i) Show that the equation of the normal to *H* at the point $P(\sec\theta, \tan\theta)$ is

$$x\sin\theta + y = 2\tan\theta.$$

(ii) This normal meets the x-axis at the point *Q*. Show that the locus of the midpoint of *PQ* as θ varies is a hyperbola. Determine its eccentricity and the coordinates of its foci. [12]

7. The function f is defined by

$$f(x)=\frac{x^3-8}{x^3-1}\ .$$

(a) Write down the equations of the asymptotes on the graph of f. [2]

(b) Find the points of intersection of the graph of f with the coordinate axes. [2]

(c) Find the coordinates of the stationary point on the graph of f and identify it as a maximum, a minimum or a point of inflection. [5]

(d) Sketch the graph of f, including the asymptotes. [3]

(e) The set *S* = [– 2, 2]. Determine

(i) f(*S*).

(ii) f^{-1}(*S*). [6]

END OF PAPER