



**GCE AS/A level**

0982/01



S16-0982-01

**MATHEMATICS – M3**

**Mechanics**

A.M. MONDAY, 27 June 2016

1 hour 30 minutes

### **ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

### **INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Take  $g$  as  $9.8 \text{ ms}^{-2}$ .

Sufficient working must be shown to demonstrate the **mathematical** method employed.

### **INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle of mass 60 kg moves along the horizontal  $x$ -axis under the action of a horizontal constant force of 1800 N. The magnitude of the resistance to motion of the particle is  $120v$  N, where  $v$  ms<sup>-1</sup> is the velocity of the particle. At time  $t = 0$  seconds, the particle is moving with velocity 8 ms<sup>-1</sup>.

- (a) Show that  $v$  satisfies the differential equation

$$\frac{dv}{dt} = 30 - 2v. \quad [2]$$

- (b) Find an expression for  $v$  at time  $t$ . Determine the limiting value of  $v$ . [7]

2. (a) A particle moves along the  $x$ -axis such that its position  $x$  m after time  $t$  seconds is given by

$$x = A \sin \omega t + B \cos \omega t.$$

Show that the motion of the particle is Simple Harmonic. State the value of  $x$  at the centre of motion and find the amplitude of the motion. [7]

- (b) Another particle moves with Simple Harmonic Motion with centre  $O$ . The particle has velocity 13 ms<sup>-1</sup> when it is 3 m from  $O$  and 5 ms<sup>-1</sup> when it is 5 m from  $O$ .

- (i) Find the period and amplitude of the motion.
- (ii) Given that the particle is at  $O$  at time  $t = 0$ , find the distance of the particle from  $O$  when  $t = 0.3$ . [9]

3. Solve the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 27t,$$

where  $x = \frac{dx}{dt} = 0$  when  $t = 0$ . Hence find the value of  $x$  when  $t = 2$ . [12]

4. A body of mass 8 kg starts from rest and falls vertically under gravity. At time  $t$  seconds, the body has fallen through a distance  $x$  metres, and its velocity is  $v$  ms<sup>-1</sup>. During the downward motion, it experiences a resisting force of  $0.4v^2$  N.

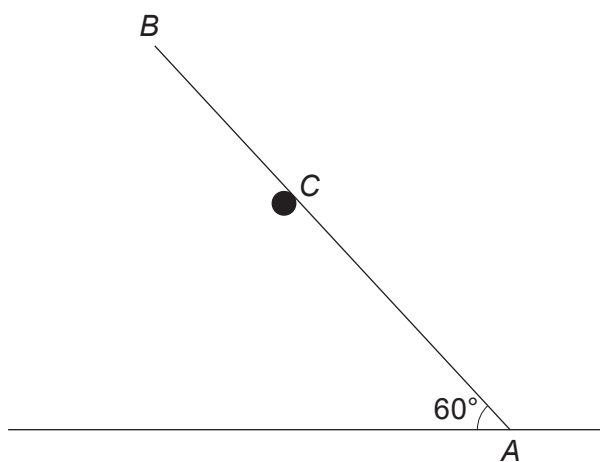
- (a) Show that  $v$  satisfies the differential equation

$$196 - v^2 = 20v \frac{dv}{dx}. \quad [2]$$

- (b) Find an expression for  $x$  in terms of  $v$  and hence calculate the value of  $x$  when the speed of the body is 10 ms<sup>-1</sup>. [6]

- (c) Find an expression for  $v$  at time  $t$  and hence find the value of  $v$  when  $t = 2$ . [8]

5. A particle  $A$ , of mass  $2\text{ kg}$ , lies on the edge of a horizontal surface. It is connected by means of a light inextensible string of length  $1.8\text{ m}$  to another particle  $B$ , of mass  $5\text{ kg}$ , which is lying on the surface  $0.2\text{ m}$  from the edge such that  $AB$  is perpendicular to the edge. The surface is at a height of  $2\text{ m}$  above the ground. Particle  $A$  is then pushed gently over the edge. Find the magnitude of the velocity with which  $B$  begins to move and the impulsive tension in the string. [8]
6. The diagram shows a uniform rod  $AB$ , of length  $10\text{ m}$  and mass  $25\text{ kg}$ , in limiting equilibrium with its end  $A$  on rough horizontal ground and point  $C$  resting against a smooth fixed peg. The rod is inclined at an angle of  $60^\circ$  to the ground.



The distance  $AC$  is  $x\text{ m}$  and the coefficient of friction between the rod and the ground is  $0.3$ .

- (a) Draw a diagram showing all the forces acting on the rod. Label all points and forces clearly. [2]
- (b) Determine the magnitude of the reaction at  $C$  and the magnitude of the normal reaction at  $A$ . [8]
- (c) Find the value of  $x$ . [4]

**END OF PAPER**