GCE AS/A Level

0974/01

S17-0974-01

MATHEMATICS – C2

Pure Mathematics

WEDNESDAY, 24 MAY 2017 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_0^2 \sqrt{7-x^2}\,dx\,.$$

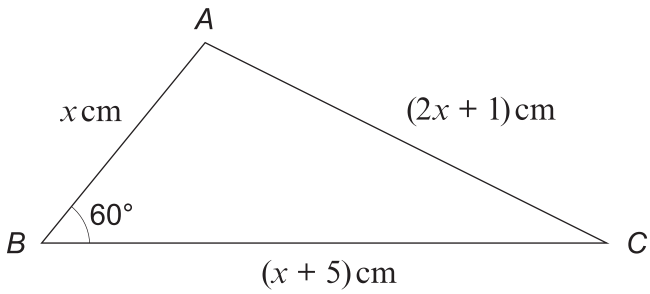
Show your working and give your answer correct to three decimal places. [4]

2. *(a)* Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$\sin^2\theta + 6\cos^2\theta + 13\sin\theta = 0.$$ [5]

(b) The angles A, B and C are the three angles of a triangle. Given that $\cos A = -0{\cdot}342$ and that $\tan(B - C) = 0{\cdot}404$, find the values of A, B and C. Give each angle correct to the nearest degree. [4]

3. The diagram below shows a sketch of the triangle *ABC* with $AB = x\,\text{cm}$, $BC = (x + 5)\,\text{cm}$, $AC = (2x + 1)\,\text{cm}$ and $A\hat{B}C = 60°$.



(a) Show that x satisfies the equation $3x^2 - x - 24 = 0$. Hence evaluate x. [4]

(b) Find the size of $A\hat{C}B$. [2]

4. *(a)* An arithmetic series has first term a and common difference d. Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right].$$ [3]

(b) The sum of the first eight terms of an arithmetic series is 156 and the sum of the first sixteen terms of the series is 760. Find the first term and common difference of this series. [4]

(c) The pth term of another arithmetic series is 2057. The $(p + 5)$th term of this series is 2102. Find the $(p + 8)$th term of the series. [3]

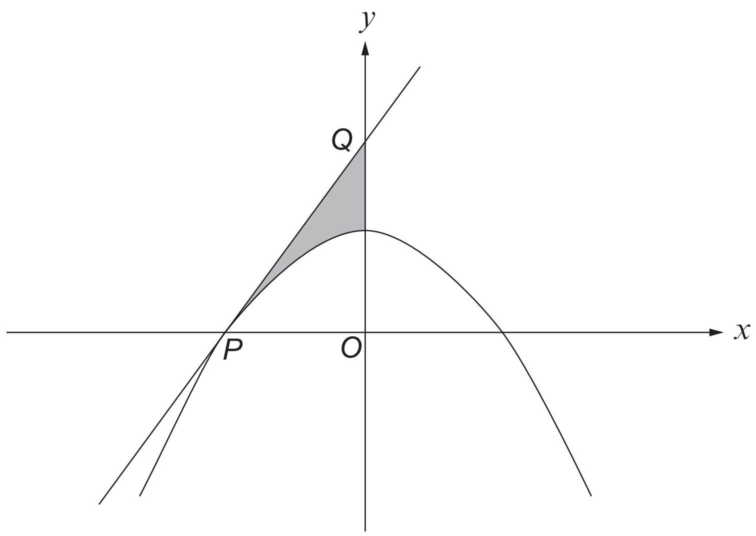
5. A rich businessman makes one donation per year to a certain charity. He starts by donating £100 in the first year. In each subsequent year, the value of the donation is 1·2 times the value of the previous year's donation.

(a) Find the value of the businessman's donation in the 12th year. Give your answer correct to the nearest pound. [2]

(b) After receiving the nth donation, the charity's treasurer calculates that over the years, the businessman has donated a **total** of £15 474, correct to the nearest pound. Find the value of n. [5]

6. *(a)* Find $\int\left(\frac{2}{x^5} - 6x^{\frac{3}{4}}\right)dx$. [2]

(b)



The diagram shows a sketch of the curve $y = 16 - x^2$ which intersects the negative x-axis at the point $P(a, 0)$.

(i) Write down the value of a.

The tangent to the curve at P intersects the y-axis at the point $Q(0, b)$.

(ii) Show that $b = 32$.

(iii) Find the area of the shaded region. [10]

7. *(a)* Given that $x > 0$, $y > 0$, show that

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y.$$ [3]

(b) Express

$$\frac{1}{3}\log_b x^{15} - \log_b 27x + 4\log_b \frac{3}{x}$$

as a single logarithm in its simplest form. [4]

(c) Given that $\log_d 5 = \frac{1}{3}$, find the value of d. [2]

8. The circle C has centre A and equation

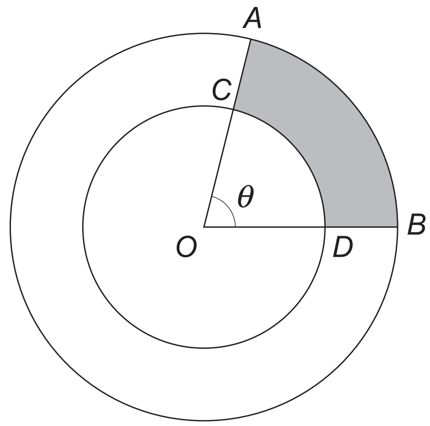
$$x^2 + y^2 + 10x - 8y + 21 = 0.$$

(a) (i) Find the coordinates of A and the radius of C.

(ii) The point P has coordinates $(-2, 0)$. Determine whether P lies inside C, on C or outside C. [5]

(b) The line L has equation $y = 2x + 4$. Show that L is a tangent to the circle C and find the coordinates of the point of contact of L and C. [5]

9.



The diagram shows two concentric circles with common centre O. The radius of the larger circle is R cm and the radius of the smaller circle is r cm. The points A and B lie on the larger circle and are such that $A\hat{O}B = \theta$ radians. The smaller circle cuts OA and OB at the points C and D respectively. The sum of the lengths of the arcs AB and CD is L cm. The area of the shaded region $ACDB$ is K cm^2.

(a) (i) Write down an expression for L in terms of R, r and θ.

(ii) Write down an expression for K in terms of R, r and θ. [2]

(b) Given that $AC = x$ cm, use your results to part *(a)* to find an expression for K in terms of x and L. [3]

10. The nth term of a number sequence is denoted by t_n. The $(n + 1)$th term of the sequence satisfies

$$t_{n+1} = 3t_n + 1,$$

for all positive integers n. Given that $t_4 = 202$,

(a) evaluate t_1, [2]

(b) explain why 29 999 999 cannot be one of the terms of this number sequence. [1]

END OF PAPER