GCE AS/A Level

0984/01

S17-0984-01

MATHEMATICS – S2
Statistics

WEDNESDAY, 14 JUNE 2017 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The independent random variables X, Y are distributed such that X is B(5, 0·4) and Y is B(8, 0·2). Given that the random variable W is defined by $W = XY$, determine

(a) the mean and the variance of W, [9]

(b) $P(W = 0)$. [4]

2. The number of computer breakdowns per day in a large IT Department may be assumed to follow a Poisson distribution with mean 0·8. In an attempt to reduce the number of breakdowns, the IT Manager moves the department to a new purpose-built office. He defines the following hypotheses

$$H_0 : \mu = 0{\cdot}8 \; ; \quad H_1 : \mu < 0{\cdot}8$$

where μ denotes the mean number of breakdowns per day after the move.
He finds that in the first 100 days after the move, there was a total of 64 computer breakdowns. You may assume that the numbers of breakdowns on successive days are independent. Calculate the approximate *p*-value of this result and interpret it in context. [7]

3. A grocer sells apples and pears. The weights of the apples may be assumed to be normally distributed with mean 110 grams and standard deviation 14 grams. The weights of the pears may be assumed to be normally distributed with mean 160 grams and standard deviation 16 grams.

(a) Find the 90th percentile of the weights of the apples. [2]

(b) George buys 10 apples. Find the probability that the total weight of his 10 apples is less than 1000 grams. [6]

(c) Sue buys 3 apples and 2 pears. Find the probability that the combined weight of her 3 apples is more than the combined weight of her 2 pears. [7]

4. A motorbike club wished to compare the fuel consumptions of two motorbike models, A and B. To do this, eight motorbikes of each model were given 15 litres of petrol and driven around a track until they ran out of petrol. The distances travelled (in miles) by the motorbikes were as follows.

Model A	168·2	170·5	164·2	169·2	165·8	166·6	162·2	168·5
Model B	161·7	166·3	167·4	164·1	162·7	160·3	165·6	163·1

You may assume that these are random samples from normal distributions with means μ_A, μ_B respectively and common standard deviation 2·5.

(a) Determine a 95% confidence interval for $\mu_A - \mu_B$. [7]

(b) Find the smallest confidence level for which the corresponding confidence interval includes zero. Give your answer as a percentage correct to three significant figures. [4]

5. Charlie is given a coin and he is told that it is biased so that the probability, p, of obtaining a head when tossed is 0·75. To test this, he defines the following hypotheses.

$$H_0: p = 0{\cdot}75; \quad H_1: p \neq 0{\cdot}75$$

(a) He decides to toss the coin 50 times and he denotes the number of heads obtained by x. He defines the following critical region.

$$(x \leqslant 31) \cup (x \geqslant 44)$$

(i) Determine the significance level of this test.

(ii) Find the probability of accepting H_0 if the value of p is actually 0·5. [7]

(b) In a further attempt to test whether or not the value of p is 0·75, he decides to toss the coin 200 times. He obtains 139 heads.

(i) Calculate the approximate p-value of this result.

(ii) Interpret the p-value. [7]

6. The continuous random variable X is uniformly distributed on the interval $[a, b]$ where $0 < a < b$.

(a) (i) Write down the probability density function of X, stating its value for all real values of x.

(ii) Hence, using integration, show that

$$E\left(X^2\right) = \frac{a^2 + ab + b^2}{3}.$$

(iii) Hence, given that $E\left(X\right) = \dfrac{a+b}{2}$, show that

$$\text{Var}\left(X\right) = \frac{(b-a)^2}{12}.$$

[8]

(b) Given that the random variable Y is defined by $Y = \dfrac{1}{X}$, $a \leqslant X \leqslant b$,

(i) determine $E(Y)$ in terms of a and b,

(ii) show that, for $\dfrac{1}{b} \leqslant y \leqslant \dfrac{1}{a}$,

$$P\left(Y \leqslant y\right) = \frac{b - \dfrac{1}{y}}{b-a}.$$

(iii) Hence find the probability density function of Y. [7]

END OF PAPER