GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - C2
0974/01

INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

Mathematics C2 May 2017

Solutions and Mark Scheme

1.

0	2·645751311		
0·5	2·598076211		
1	2·449489743		
1·5	2·179449472		
2	1·732050808	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0·5$ M1

$I \approx \frac{0·5}{2} \times \{2·645751311 + 1·732050808 + 2(2·598076211 + 2·449489743 + 2·179449472)\}$

$I \approx 18·83183297 \times 0·5 \div 2$

$I \approx 4·707958243$

$I \approx 4·708$ (f.t. one slip) A1

Special case for candidates who put $h = 0·4$

0	2·645751311		
0·4	2·615339366		
0·8	2·521904043		
1·2	2·357965225		
1·6	2·107130751		
2	1·732050808	(all values correct)	(B1)

Correct formula with $h = 0·4$ (M1)

$I \approx \frac{0·4}{2} \times \{2·645751311 + 1·732050808 + 2(2·615339366 + 2·521904043 + 2·357965225 + 2·107130751)\}$

$I \approx 23·58248089 \times 0·4 \div 2$

$I \approx 4·716496178$

$I \approx 4·716$ (f.t. one slip) (A1)

Note: Answer only with no working shown earns 0 marks

2. (*a*) $\sin^2\theta + 6(1 - \sin^2\theta) + 13\sin\theta = 0,$

(correct use of $\cos^2\theta = 1 - \sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant m1

$5\sin^2\theta - 13\sin\theta - 6 = 0 \Rightarrow (5\sin\theta + 2)(\sin\theta - 3) = 0$

$\Rightarrow \sin\theta = -\frac{2}{5}$, ($\sin\theta = 3$) (c.a.o.) A1

$\theta = 203{\cdot}58°, 336{\cdot}42°$ B1 B1

Note: Subtract (from final two marks) 1 mark for each additional root in range from $5\sin\theta + 2 = 0$, ignore roots outside range.
$\sin\theta = -$, f.t. for 2 marks, $\sin\theta = +$, f.t. for 1 mark

(*b*) $A = 110°$ B1

$B - C = 22°$ B1

$110° + B + C = 180°$ (f.t. candidate's value for A) M1

$B = 46°, C = 24°$ (f.t. one error) A1

3. (*a*) $(2x + 1)^2 = x^2 + (x + 5)^2 - 2 \times x \times (x + 5) \times \cos 60°$ (o.e.)

(correct use of cos rule) M1

$3x^2 - x - 24 = 0$ (convincing) A1

An attempt to solve the given quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(ax + b)(cx + d)$, with $a \times c = 3$ and $b \times d = -24$ M1

$(3x + 8)(x - 3) = 0 \Rightarrow x = 3$ A1

(*b*) $\frac{\sin ACB}{3} = \frac{\sin 60°}{7}$

(substituting the correct values in the correct places in the sin rule) M1

$ACB = 21{\cdot}8°$ A1

(Allow ft for x>0 obtained in (a) for M1A1)

4. (*a*) $S_n = a + [a + d] + \ldots + [a + (n-1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n-1)d] + [a + (n-2)d] + \ldots + a$

In order to make further progress, the two expressions for S_n must contain at least three pairs of terms, including the first pair, the last pair and one other pair of terms

Either:

$2S_n = [a + a + (n-1)d] + [a + a + (n-1)d] + \ldots + [a + a + (n-1)d]$

Or:

$2S_n = [a + a + (n-1)d]$ n times M1

$2S_n = n[2a + (n-1)d]$

$S_n = \frac{n}{2}[2a + (n-1)d]$ (convincing) A1

(*b*) $\frac{8}{2} \times (2a + 7d) = 156$ B1

$2a + 7d = 39$

$\frac{16}{2} \times (2a + 15d) = 760$ B1

$2a + 15d = 95$

An attempt to solve the candidate's two derived linear equations simultaneously by eliminating one unknown M1

$d = 7$, $a = -5$ (c.a.o.) A1

(*c*) $d = 9$ B1

A correct method for finding $(p + 8)$ th term M1

$(p + 8)$ th term = 2129 (c.a.o.) A1

5. (*a*) $a = 100$, $r = 1{\cdot}2$

Value of donation in 12th year = $100 \times 1{\cdot}2^{11}$ M1

Value of donation in 12th year = £743 A1

(*b*) $100 \times \frac{(1 - 1{\cdot}2^n)}{1 - 1{\cdot}2} = 15474$ M1

$1 - 1{\cdot}2^n = 154{\cdot}74 \times (-0{\cdot}2)$ m1

$1{\cdot}2^n = 31{\cdot}948$ A1

$n = \frac{\log 31{\cdot}948}{\log 1{\cdot}2}$ m1

$n = 19$ cao A1

6. (*a*) $2 \times \dfrac{x^{-4}}{-4} - 6 \times \dfrac{x^{7/4}}{7/4} + c$ B1, B1

(–1 if no constant term present)

(*b*) (i) $16 - a^2 = 0 \Rightarrow -4$ B1

(ii) $\dfrac{dy}{dx} = -2x$ M1

Gradient of tangent = 8 (f.t. candidate's value for a) A1

$b = 32$ (convincing) A1

(iii) Use of integration to find the area under the curve M1

$\int (16 - x^2)\, dx = 16x - (1/3)x^3$ (correct integration) A1

Correct method of substitution of candidate's limits m1

$$\left[16x - (1/3)x^3\right]_{-4}^{0} = 0 - [-64 - (-64/3)] = 128/3$$

Area of the triangle = 64 (f.t. candidate's value for a) B1

Use of candidate's value for a and 0 as limits and trying to find total area by subtracting area under curve from area of triangle m1

Shaded area = 64 – 128/3 = 64/3 (c.a.o.) A1

7. (*a*) Let $p = \log_a x$, $q = \log_a y$

Then $x = a^p$, $y = a^q$ (the relationship between log and power) B1

$\dfrac{x}{y} = \dfrac{a^p}{a^q} = a^{p-q}$ (the laws of indicies) B1

$\log_a x/y = p - q$ (the relationship between log and power)

$\log_a x/y = p - q = \log_a x - \log_a y$ (convincing) B1

(*b*) $\dfrac{1}{3}\log_b x^{15} = \log_b x^5$, $4\log_b 3/x = \log_b 3^4/x^4$

(one correct use of power law) B1

$\dfrac{1}{3}\log_b x^{15} - \log_b 27x + 4\log_b 3/x = \log_b \dfrac{x^5 \times 3^4}{27x \times x^4}$ (addition law) B1

(subtraction law) B1

$\dfrac{1}{3}\log_b x^{15} - \log_b 27x + 4\log_b 3/x = \log_b 3$ (c.a.o.) B1

(*c*) $\log_d 5 = \dfrac{1}{3} \Rightarrow 5 = d^{1/3}$

(rewriting log equation as power equation) M1

$d = 125$ A1

8.	(*a*)	(i)	$A(-5, 4)$	B1
			A correct method for finding radius	M1
			Radius = $\sqrt{20}$	A1
		(ii)	**Either:**	
			A correct method for finding AP^2	M1
			$AP^2 = 25\ (> 20) \Rightarrow P$ is outside C	
			(f.t. candidate's coordinates for A)	A1
			Or:	
			An attempt to substitute $x = -2$, $y = 0$ in the equation of C	(M1)
			$(-2)^2 + 0^2 + 10 \times (-2) - 8 \times 0 + 21 = 5\ (> 0)$	
			$\Rightarrow P$ is outside C	(A1)
	(*b*)		An attempt to substitute $(2x + 4)$ for y in the equation of the circle	M1
			$5x^2 + 10x + 5 = 0$	A1
		Either:	Use of $b^2 - 4ac$	m1
			Discriminant = 0, $\Rightarrow y = 2x + 4$ is a tangent to the circle	A1
			$x = -1$, $y = 2$	A1
		Or:	An attempt to factorise candidate's quadratic	(m1)
			Repeated (single) root, $\Rightarrow y = 2x + 4$ is a tangent to the circle	(A1)
			$x = -1$, $y = 2$	(A1)

9.	(*a*)	(i)	$L = R\theta + r\theta$	B1
		(ii)	$K = \frac{1}{2}R^2\theta - \frac{1}{2}r^2\theta$	B1
	(*b*)		$K = \frac{1}{2}\theta(R + r)(R - r)$	M1
			$L = \theta(R + r)$, $R - r = x$ (both expressions)	m1
			$K = \frac{1}{2}Lx$	A1

<u>Alternative solution</u>

$K = \frac{1}{2}\theta(R^2 - r^2)$

$K = \frac{1}{2}\theta((r+x)^2 - r^2)$ (M1)

$K = \frac{1}{2}\theta(2rx + x^2)$

$K = \frac{1}{2}x\theta(2r + x)$ (m1)

$K = \frac{1}{2}x\theta(R + r)$

$K = \frac{1}{2}Lx$ (A1)

10.	(*a*)	$t_3 = 67$		B1
		$t_1 = 7$	(f.t. candidate's value for t_3)	B1
	(*b*)	29999999 is of the form $3k - 1$ (not $3k + 1$)	(o.e.)	
		OR		
		The number does not end in a 2 or a 7		E1