GCE AS MARKING SCHEME

SUMMER 2018

AS (NEW)
MATHEMATICS – UNIT 1 PURE MATHEMATICS A
2300U10-1

INTRODUCTION

This marking scheme was used by WJEC for the 2018 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

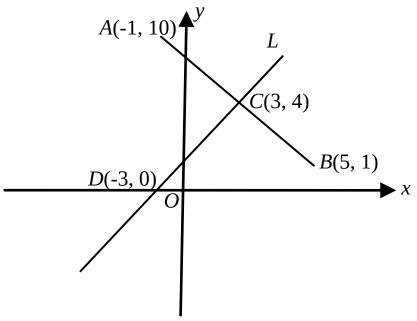
It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

AS Unit 1 Pure Mathematics A

SUMMER 2018 MARK SCHEME

Q	Solution	Mark	Notes
1(a)	$\dfrac{24\sqrt{a}}{\left(\sqrt{a}+3\right)^2-\left(\sqrt{a}-3\right)^2}$		
	$=\dfrac{24\sqrt{a}}{\left[\left(\sqrt{a}+3\right)+\left(\sqrt{a}-3\right)\right]\left[\left(\sqrt{a}+3\right)-\left(\sqrt{a}-3\right)\right]}$	M1	factorisation x^2-y^2
	Or $\dfrac{24\sqrt{a}}{(a+6\sqrt{a}+9)-(a-6\sqrt{a}+9)}$	(M1)	one correct expansion
	$=\dfrac{24\sqrt{a}}{\left(2\sqrt{a}\right)(6)}$	A1	si correct simplified denominator.
	$= 2$	A1	cao
1(b)	$\dfrac{(3\sqrt{7}+5\sqrt{3})(\sqrt{7}-\sqrt{3})}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})}$	M1	
	$=\dfrac{3\times7-3\sqrt{7}\sqrt{3}+5\sqrt{7}\sqrt{3}-5\times3}{7-\sqrt{7}\sqrt{3}+\sqrt{7}\sqrt{3}-3}$	A1	numerator correct
		A1	denominator correct
	$=\dfrac{21-3\sqrt{21}+5\sqrt{21}-15}{7-3}$		
	$=\dfrac{1}{2}(3+\sqrt{21})$	A1	oe. cao

Q	Solution	Mark	Notes
2(a)			
	Grad AB = $\frac{1-10}{5+1} = \frac{-9}{6} = -\frac{3}{2}$	B1	
	Correct method for finding the equ AB	M1	
	Equ AB is $y - 1 = -\frac{3}{2}(x - 5)$	A1	ft grad AB
	OR Equ AB is $y - 10 = -\frac{3}{2}(x - (-1))$	(A1)	ft grad AB
	$2y - 2 = -3x + 15$		
	$2y + 3x = 17$		
	L and AB meet when:		
	$4x - 6y = -12$		
	$9x + 6y = 51$		
	$13x = 39$	m1	ft eqns one variable eliminated
	$x = 3$, $y = 4$	A1	cao
2(b)	$AC : CB = 3-(-1) : 5-3$	M1	oe (10-4):(4-1)
	$AC : CB = 4 : 2 = 2 : 1$	A1	ft coordinates C

Accept unsimplified values

2(c)	D is the point (-3, 0)	B1	

Q	Solution	Mark	Notes
2(d)(i)	*AB* is perpendicular to *DC*, because		
	grad CA × grad DC = $-\frac{3}{2} \times \frac{2}{3} = -1$		
	Hence *L* is perpendicular to *AB*.	B1	needs some evidence,
			not just $-\frac{3}{2} \times \frac{2}{3} = -1$
2(d)(ii)	Correct method for finding distance	M1	
	$CA = \sqrt{(10-4)^2+(3+1)^2} = \sqrt{52}$	A1	may be seen in (b)
			ft coordinates *C*
	$DC = \sqrt{(4-0)^2+(3+3)^2} = \sqrt{52}$	A1	ft coordinates *C* but not *D*
	Area of triangle *ACD* = $\frac{1}{2} \times CA \times DC$	M1	used
	Area = $\frac{1}{2} \times \sqrt{52} \times \sqrt{52}$		
	Area = 26	A1	cao

Q	Solution	Mark	Notes
3	$2 - 3(1 - \sin^2\theta) = 2\sin\theta$	M1	subt for $\cos^2$
	$3\sin^2\theta - 2\sin\theta - 1 = 0$		
	$(3\sin\theta + 1)(\sin\theta - 1) = 0$	m1	allow $(3\sin\theta - 1)(\sin\theta + 1)$
	$\sin\theta = 1, -\frac{1}{3}$	A1	cao
	$\sin\theta = 1$, $\theta = 90°$	B1	
	$\sin\theta = -\frac{1}{3}$, $\theta = 199.47°, 340.53°$	B1	one correct angle
		B1	second correct angle

Use of quadratic formula only earns m1 if correct substitution seen to have been made, or implied by the right answers being obtained.

Ignore all solutions outside required range.

Full follow through for one positive and one negative value for $\sin\theta>0$ for B1 and $\sin\theta<0$ for B1 for one correct value and B1 for a second correct value.

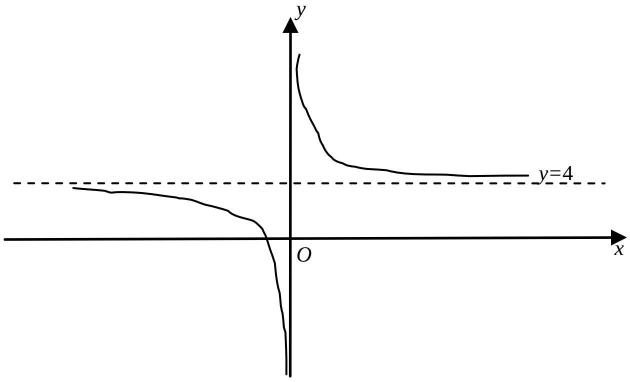
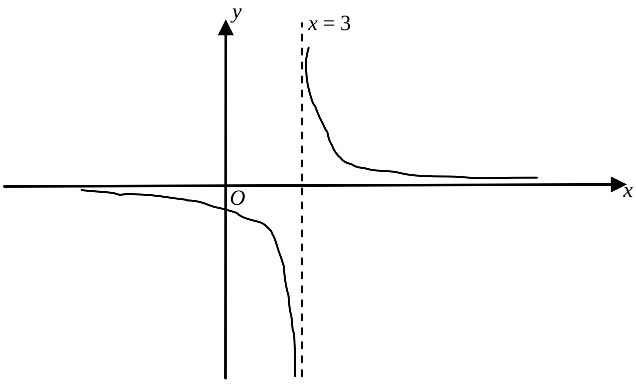
Two negative values for $\sin\theta$, award B1 B1 for one pair of correct solutions, ignore other pair even if incorrect. Award B1 for only one correct solution.

Two positive values for $\sin\theta$, award B1 for one pair of correct solutions, ignore other pair even if incorrect.

Q	Solution	Mark	Notes
4(a)	$y = 5x^{-1} + 6x^{\frac{1}{3}}$		
	$\dfrac{dy}{dx} = -5x^{-2} + 6\times\dfrac{1}{3}x^{-\frac{2}{3}} = -\dfrac{5}{x^2} + 2x^{-\frac{2}{3}}$	B1	one correct differentiation
		B1	2nd correct differentiation
	When $x = 8$,		
	$\dfrac{dy}{dx} = -\dfrac{5}{64} + 2\times\dfrac{1}{4} = \dfrac{27}{64}$ (=0.42(1875))	B1	cao
4(b)	$\int 5x^{\frac{3}{2}} + 12x^{-5} + 7\,dx$		
	$= 5\times\dfrac{x^{\frac{5}{2}}}{\frac{5}{2}} + 12\times\dfrac{1}{-4}x^{-4} + 7x + C$	B1	one correct integration
		B1	a second correct integration
		B1	all correct including C
	$\int 5x^{\frac{3}{2}} + 12x^{-5} + 7\,dx$ ⍰ $= 2x^{\frac{5}{2}} - 3x^{-4} + 7x + C$		

Award B1 once correct differentiation/integration seen, index simplified. Ignore subsequent work.

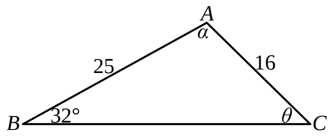
Ignore presence of integral sign after terms integrated.

Q	Solution	Mark	Notes
5(a)			
		B1	correct curve (moved up)
		B1	y =4 and x=0 as asymptotes
5(b)			
		B1	correct curve (moved to right)
		B1	x = 3 and y=0 as asymptotes

Q	Solution	Mark	Notes
6(a)	$x + 2 = 14 + 5x - x^2$	M1	
	$x^2 - 4x - 12 = 0$		
	$(x + 2)(x - 6) = 0$	m1	si $(x+a)(x+b)=0$ if ab=their constant
	$x = -2$, $y = 0$	A1	or $x = -2, 6$
	$A(-2, 0)$		
	$x = 6$, $y = 8$	A1	or $y = 0, 8$
	$B(6, 8)$		
	SC $14 + 5x - x^2 = 0$ M1, $x = -2$, $y = 0$ A1		
	SC $x + 2 = 0$, M1, , $x = -2$, $y = 0$ A1		
6(b)	$A = \int_{-2}^{6} 14 + 5x - x^2 \mathrm{d}x$	M1	limits not required,
			must be sure integrating
	$A = \left[14x + \frac{5}{2}x^2 - \frac{x^3}{3}\right]_{-2}^{6}$	B1	correct integration of
			quadratic expression
	$A = \left[102 - \left(-\frac{46}{3}\right)\right] = \frac{352}{3}$ $(= 117\frac{1}{3})$	m1	correct use of limits
	Area of triangle = 0.5 × 8 × 8 = 32	B1	si ft coordinate of B, not (7, 0)
	Required area = $\frac{352}{3}$ - 32	m1	
	Required area = $\frac{256}{3} = 85\frac{1}{3}$	A1	cso supported by working

Q	Solution	Mark	Notes
7	$\dfrac{\sin^3\theta + \sin\theta\cos^2\theta}{\cos\theta}$		
	$\equiv \dfrac{\sin\theta(\sin^2\theta + \cos^2\theta)}{\cos\theta}$	B1	or substitute for $\cos^2\theta/\sin^2\theta$
			$\dfrac{\sin\theta(\sin^2\theta) + \sin\theta\cos^2\theta}{\cos\theta}$
	$\dfrac{\sin\theta}{\cos\theta}$	B1	simplifying numerator
	$\equiv \tan\theta$	B1	sin/cos = tan
			Withhold last mark if proof not mathematical.

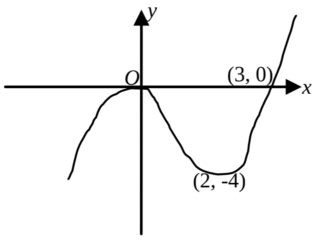
Q	Solution	Mark	Notes
8(a)	Use factor th^{m} with $f(x)=2x^3+px^2+qx–12$	M1	x=2 or -2
	$2(2)^3+p(2)^2+q(2)–12=16+4p+2q-12=0$		
	$2(-2)^3+p(-2)^2+q(-2)–12=-16+4p-2q-12=0$	A1	either equation
	$2p + q = -2$		
	$2p – q = 14$		
	Adding $4p = 12$	m1	ft linear equations
	$p = 3$		
	$q = -8$	A1	cao both values
8(b)	Other factor is $(2x + 3)$	B1	sight of $(2x + 3)$
OR			
8(a)	$2x^3+px^2+qx–12 = (x + 2)(x - 2)(ax + b)$	(M1)	
	$2x^3+px^2+qx–12 = (x^2 - 4)(2x + 3)$		
	$2x^3+px^2+qx–12 = 2x^3+3x^2-8x–12$	(A1)	
	Compare coefficients	(m1)	
	$p = 3$		
	$q = -8$	(A1)	cao both values
8(b)	Other factor is $(2x + 3)$	(B1)	may be seen in (a)

Q	Solution	Mark	Notes
9			
	sine rule: $\dfrac{16}{\sin 32^\circ} = \dfrac{25}{\sin\theta}$	M1	
	θ = 55.8937° or 124.1063°	A1	both, accept 56, 124
	α = 92.1063° or 23.8937°	m1	either value
	Required area = $\frac{1}{2}\times 25\times 16(\sin 92.1063^\circ)$	m1	use of $\frac{1}{2}bc\sin A$
	= 199.86 (cm^2)		
	or Required area = $\frac{1}{2}\times 25\times 16(\sin 23.8937^\circ)$		
	= 81.01 (cm^2)	A1	both areas correct
			accept answers rounding to 200, 81
	OR		
	$16^2 = 25^2 + x^2 - 2\times 25\times x\times\cos 32^\circ$	(M1)	
	$x^2 - 42.4024x + 369 = 0$		
	$x = \frac{1}{2}(42.4024 \pm \sqrt{42.4024^2 - 4\times 369})$	(m1)	
	x = 30.1729, 12.2295	(A1)	both values, accept answers rounding to 30, 12
	Area = $\frac{1}{2}ac\sin B$	(m1)	used
	Area = 199.86, 81.01	(A1)	accept answers rounding to 200, 81

Q	Solution	Mark	Notes
10(a)	$\left(a+\sqrt{b}\right)^4 = a^4 + 4a^3(\sqrt{b}) + 6a^2(\sqrt{b})^2$		
	$+4a(\sqrt{b})^3 + (\sqrt{b})^4$	B1	at least 3 correct terms
		B1	all terms correct.
	$\left(a+\sqrt{b}\right)^4 = a^4+4a^3\sqrt{b}+6a^2b+4ab\sqrt{b}+b^2$		
10(b)	$\left(a-\sqrt{b}\right)^4 = a^4 - 4a^3(\sqrt{b}) + 6a^2(\sqrt{b})^2$		
	$-4a(\sqrt{b})^3 + (\sqrt{b})^4$	M1	change of sign
	$\left(a+\sqrt{b}\right)^4 + \left(a-\sqrt{b}\right)^4 = 2a^4+12a^2b+2b^2$	A1	cao, b's simplified

Q	Solution	Mark	Notes
11(a)	$\|\mathbf{u}\| = \sqrt{9^2 + (-40)^2}$	M1	method for length
	$\|\mathbf{u}\| = 41$		
	$\|\mathbf{v}\| = \sqrt{3^2 + (-4)^2}$		
	$\|\mathbf{v}\| = 5$	A1	either correct
	$\mu\|\mathbf{v}\| > \|\mathbf{u}\|$ if $5\mu > 41$		
	$\mu > 8.2$	A1	A0 for =
11(b)	$AC : CB = 2 : 3$		
	$3\mathbf{AC} = 2\mathbf{CB}$	M1	si any correct method
	$3(\mathbf{c} - \mathbf{a}) = 2(\mathbf{b} - \mathbf{c})$		
	C has position vector $\mathbf{c} = \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$	A1	
	$\mathbf{c} = \frac{3}{5}(11\mathbf{i} - 4\mathbf{j}) + \frac{2}{5}(21\mathbf{i} + \mathbf{j})$		
	$\mathbf{c} = \frac{1}{5}[(33 + 42)\mathbf{i} + (-12 + 2)\mathbf{j}]$		
	$\mathbf{c} = 15\mathbf{i} - 2\mathbf{j}$	A1	cao
	OR		
	$\mathbf{AB} = 10\mathbf{i} + 5\mathbf{j}$ / $\mathbf{BA} = -10\mathbf{i} - 5\mathbf{j}$	(B1)	
	$\mathbf{c} = (11\mathbf{i} - 4\mathbf{j}) + \frac{2}{5}(10\mathbf{i} + 5\mathbf{j})$		
	or		
	$\mathbf{c} = (21\mathbf{i} + \mathbf{j}) - \frac{3}{5}(10\mathbf{i} + 5\mathbf{j})$	(M1)	
	$\mathbf{c} = 15\mathbf{i} - 2\mathbf{j}$	(A1)	cao

Q	Solution	Mark	Notes
12	$4x^2 + 8x - 8 = m(4x - 3)$		
	$4x^2 + (8 - 4m)x + (3m - 8) = 0$	M1	terms grouped, brackets not required
	Discriminant = $(8 - 4m)^2 - 4\times4(3m - 8)$	m1	ft equivalent difficulty
	If real roots, then discriminant ≥ 0	m1	accept >
	$(2 - m)^2 - (3m - 8) \geq 0$		
	$m^2 - 7m + 12 \geq 0$	A1	cao write as quadratic inequality
	$(m - 3)(m - 4) \geq 0$		
	$m \leq 3$ or $m \geq 4$	A1	cao , or, union A0 for and, strict inequality

Q	Solution	Mark	Notes
13(a)	$\dfrac{dy}{dx} = 3x^2 - 6x$	B1	
	At stationary points $\dfrac{dy}{dx} = 0$.	M1	si
	$3x(x - 2) = 0$		
	$x = 0$, $x = 2$	A1	any pair of correct values
	$y = 0$, $y = -4$	A1	all 4 values correct
	$\dfrac{d^2y}{dx^2} = 6x - 6$	M1	oe ft quadratic dy/dx
	$x = 0$, $\dfrac{d^2y}{dx^2} = -6 < 0$.		
	(0, 0) is a maximum point	A1	ft their x value
	$x = 2$, $\dfrac{d^2y}{dx^2} = 6 > 0$.		
	(2, -4) is a minimum point	A1	ft their x value provided different conclusion
13(b)		M1	shape for +ve cubic
		A1	(3, 0)
		A1	(0, 0) max, (2, -4) ft min pt
13(c)	The integral is negative since $y \leq 0$ in the relevant interval.	B1	

Q	Solution	Mark	Notes
14(a)	Statement A is false.		
	Let $c = 2$, $d = 1$	M1	$d \neq 0$, $d \neq 2c$
	LHS = $(2\times 2 - 1)^2 = 9$		
	RHS = $4\times 2^2 - 1 = 15$		
	Therefore LHS ≠ RHS	A1	correct verification
14(b)	Statement B is true		
	RHS = $(2c - d)(4c^2 + 2cd + d^2)$		
	$= 8c^3 + 4c^2d + 2cd^2 - 4c^2d - 2cd^2 - d^3$	M1	correct removal of brackets attempted
		A1	algebra all correct
	$= 8c^3 - d^3$ = LHS		answer given

Q	Solution	Mark	Notes
15	$V = Ae^{kt}$		Given
	When $t = 0$, $V = 30000$	M1	use of either condition
	$A = 30000$	A1	si
	When $t = 2$, $V = 20000$		
	$e^{2k} = \frac{2}{3}$	A1	
	When $t = 6$, $V = 30000e^{6k}$	m1	
	$V = 30000(e^{2k})^3$	A1	oe,
	$V = 8889$		
	$V = 8900$	A1	cao
	OR		
	$2k = \ln(\frac{2}{3})$ (= -0.405….)	(A1)	
	$k = -0.203….$		
	$V = 30000e^{-0.203\ldots\times 6}$	(m1)	
	$V = 8900$	(A1)	cao

Q	Solution	Mark	Notes
16	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 13 - 4x$	M1	
	$13 – 4x = 1$	m1	
	$x = 3$	A1	cao
	$y = 7 + 13×3 -2×3^2 = 28$	A1	cao
	Equation of tangent is $y = x + c$		
	$28 = 3 + c$		
	$c = 25$	A1	ft derived x and y
	Equation of tangent is $y = x + 25$		
	OR		
	Curve and line meets when		
	$7 + 13x -2x^2 = x + c$		
	$2x^2 – 12x + (c – 7) = 0$	(M1)	
	Line is a tangent if discriminant = 0		
	$(-12)^2 – 4 × 2(c – 7) = 0$	(m1)	
	$c = 25$	(A1)	cao
	$7 + 13x -2x^2 = x + 25$		
	$x^2 – 6x + 9 = 0$		
	$x = 3$	(A1)	cao
	$y = 28$	(A1)	ft derived x and c

Q	Solution	Mark	Notes
17(a)	$\log_{10}x^2 - \log_{10}5 + \log_{10}2 = 1$	B1	one use of laws of logs
	$\log_{10}\left(\dfrac{2x^2}{5}\right) = 1$	B1	one use of different law of logs
	$\dfrac{2x^2}{5} = 10$	B1	logs removed
	$x^2 = 25$		
	$x = 5$	B1	cao (B0 for $x = \pm5$)
	OR		
	$2\log_{10}x = 1.39794….$	(B1)	
	$\log_{10}x = 0.69897….$	(B1)	
	$x = 10^{0.69897….}$	(B1)	
	$x = 5$	(B1)	B0 if there is evidence premature approximation
17(b)	$e^{0.5x} = 1.5$		
	$0.5x = \ln(1.5)$	M1	
	$x = 2\ln(1.5) = 0.81(093)$	A1	
17(c)	$2^{2x} - 10\times2^x = y^2 – 10y$	B1	
	$y^2 – 10y + 16 = 0$	M1	
	$(y – 2)(y – 8) = 0$		
	$y = 2, 8$	A1	
	$2^x = 2, 8$	m1	
	$x = 1, 3$	A1	

Q	Solution	Mark	Notes
18(a)	Grad of AB = $\frac{6-5}{4-(-3)} = \frac{1}{7}$	M1	method for gradient
	Grad of AC = $\frac{6-(-1)}{4-5} = -7$	A1	either correct
	Hence Grad of AB× Grad of AC = -1		
	AB is perpendicular to AC		
	Hence $B\hat{A}C$ is a right angle	A1	
	OR		
	$AB^2 = 1^2 + 7^2 = 50$	(M1)	At least one correct
	$BC^2 = 8^2 + 6^2 = 100$		
	$AC^2 = 1^2 + 7^2 = 50$	(A1)	all three correct
	$BC^2 = AB^2 + BC^2$		
	Hence $B\hat{A}C$ is a right angle	(A1)	
	OR		
	$\cos A = \frac{50+50-100}{2\sqrt{50}\sqrt{50}}$ = 0, hence A=90°	(A1)	

Q	Solution	Mark	Notes
18(b)	Centre of circle is midpoint of *BC*	M1	
	Centre of circle = $\left(\frac{-3+5}{2}, \frac{5-1}{2}\right)$		
	Centre of circle = (1, 2)	A1	
	Radius = $\frac{1}{2}\sqrt{(5-(-3))^2+(-1-5)^2}$	M1	may be seen in (a)
	Radius = 5	A1	
	Equ of circle is $(x-1)^2 + (y-2)^2 = 5^2$	A1	ft centre and radius, isw
			One must be correct
	$x^2 + y^2 - 2x - 4y - 20 = 0$		
	OR		
	Equ of circle is $x^2 + y^2 + ax + by + c = 0$	(M1)	
	At $A(4, 6)$ $4a + 6b + c = -52$	(A1)	one correct equation
	At $B(-3, 5)$ $-3a + 5b + c = -34$		
	At $C(5, -1)$ $5a - b + c = -26$	(A1)	All 3 equations correct
	Solving simultaneously	(m1)	any correct method
	$7a + b = -18$		
	$-a + 7b = -26$		
	$7a + b = -18$		
	$-7a + 49b = -182$		
	$50b = -200$		
	$b = -4$, $a = -2$, $c = -20$	(A1)	all 3 values correct
	Equ of circle is:		
	$x^2 + y^2 - 2x - 4y - 20 = 0$		