GCE A LEVEL MARKING SCHEME

SUMMER 2022

A LEVEL (NEW)
MATHEMATICS
UNIT 3 PURE MATHEMATICS B
1300U30-1

INTRODUCTION

This marking scheme was used by WJEC for the 2022 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE A LEVEL MATHEMATICS

UNIT 3 PURE MATHEMATICS B

SUMMER 2022 MARK SCHEME

Q	Solution	Mark	Notes
1	$6(1 + \tan^2 x) - 8 = \tan x$	M1	use of $\sec^2 x = 1 + \tan^2 x$ Must be seen for M1
	$a\tan^2 x + b\tan x + c = 0$		
	$6\tan^2 x - \tan x - 2 = 0$		
	$(A\tan x + B)(C\tan x + D) = 0$	m1	$AC = a$ and $BD = c$, $c \neq 0$ oe
	$(3\tan x - 2)(2\tan x + 1) = 0$		
	$\tan x = -\frac{1}{2}, \frac{2}{3}$	A1	cao
	$\tan x = \frac{2}{3}$, $x = 33.69°, 213.69°$	B1	first 2 correct solutions Condone 0.588^c, 3.730^c
	$\tan x = -\frac{1}{2}$, $x = 153.43°$,	B1	3rd correct solution Condone 2.678^c
	$x = 333.43°$	B1	4th correct solution Condone 5.820^c

Notes: If one or two roots obtained for tan x, even if incorrectly obtained, full follow through from these values for B1 B1 B1, provided one +ve and one -ve root. If only one sign obtained, only B1 available for one pair of correct angles.

Do not follow through for sin, cos or anything else.

Ignore all roots outside range $0° \leq x \leq 360°$.

For 5th, 6th, 7th extra root within range, -1 mark each extra root.

If all answers in radians, but radians **not** specified, penalise -1.

Accept all answers correctly rounded to the nearest whole number or better.

Q	Solution	Mark	Notes
2(a)	$y = x^3\ln(5x)$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2\ln(5x) + x^3\frac{5}{5x}$	M1	$f(x)\ln(5x) + x^3g(x)$
			M0 if $f(x) = 0$ or1 or $g(x) = 0$ or 1
		A1	$3x^2\ln(5x)$
		A1	$x^3\frac{5}{5x}$
			ISW
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2\ln(5x) + x^2 = x^2(3\ln(5x) + 1)$		
2(b)	$y = (x + \cos 3x)^4$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4(x + \cos 3x)^3(1 - 3\sin 3x)$	M1	$4(x + \cos 3x)^3\, f(x)$
			M0 if $f(x) = 1$
		A1	$f(x) = (1 - 3\sin 3x)$
			Condone absence of brackets
			for M1 A0, unless corrected for A1.
			ISW

Q	Solution	Mark	Notes
3	$OB\left(=\frac{4}{\cos\frac{\pi}{3}}\right) = 8$ or $OA\ \left(=\frac{4}{\tan 30°}\right) = 4\sqrt{3}$	B1	si (OA = 6.928….)
	Area $OAB = \frac{1}{2} \times 4 \times 8\sin\frac{\pi}{3}$	M1	Use of $A = \frac{1}{2} \times AB \times OA$
	$= 8\sqrt{3} = 13.856\ldots$		
	Area $OBC = \frac{1}{2} \times 8 \times 8 \times \frac{\pi}{3}$	M1	Use of $A = \frac{1}{2}r^2\theta$
			Or $A = \frac{1}{6}\pi r^2$
	$= \frac{32\pi}{3} = 33.510\ldots$		ft OB, OA
	Required area $OABC$ = 47.37 (m^2)	A1	cao Must be to 2dp

Q	Solution	Mark	Notes
4	$\frac{a}{1-r} = 120$	B1	si
	$\frac{a}{1-4r^2} = 112\frac{1}{2}$	B1	si
	$120(1 - r) = \frac{225}{2}(1 - 4r^2)$	M1	or elimination of r
	$900r^2 - 240r + 15 = 0$ or $a^2 - 208a + 10800 = 0$	m1	attempt to solve their quadratic equation Implied by correct answers
	$60r^2 - 16r + 1 = 0$		
	$(6r - 1)(10r - 1) = 0$		
	$r = \frac{1}{6}$, $r = \frac{1}{10}$	A1	One correct pair, cao
	$a = 100$, $a = 108$	A1	all correct, cao

Q	Solution	Mark	Notes
5(a)	$\left(\frac{6x+4}{(x-1)(x+1)(2x+3)}=\right)\ \frac{A}{(x-1)}+\frac{B}{(x+1)}+\frac{C}{(2x+3)}$	M1	correct form
			Implied by equation below
	$6x + 4 = A(x + 1)(2x + 3) + B(x - 1)(2x + 3)$		
	$+ C(x + 1)(x - 1)$	M1	si correct equation
	Put $x = -1$, $-2 = B(-2)(1)$		
	$B = 1$		
	Put $x = -\frac{3}{2}$, $-9 + 4 = C(-\frac{1}{2})(-\frac{5}{2})$		
	$C = -4$	A1	two correct constants
	Put $x = 1$, $10 = A(2)(5)$		
	$A = 1$	A1	third constant correct
	$f(x)=\frac{1}{(x-1)}+\frac{1}{(x+1)}-\frac{4}{(2x+3)}$		
5(b)	$\int\frac{3x+2}{(x-1)(x+1)(2x+3)}\,\mathrm{d}x$		
	$=\int\frac{1}{2}\left[\frac{1}{(x-1)}+\frac{1}{(x+1)}-\frac{4}{(2x+3)}\right]\mathrm{d}x$		
	$=\frac{1}{2}[\ln\lvert x-1\rvert+\ln\lvert x+1\rvert-2\ln\lvert 2x+3\rvert\ (+\ln C)]$	B3	B1 correct int of $\frac{1}{(x-1)}$
			B1 correct int of $\frac{1}{(x+1)}$
			B1 correct int of $\frac{K}{(2x+3)}$
			Condone no modulus signs for B3
		M1	attempt to tidy up into one ln term M0 if extra terms seen
	$=\frac{1}{2}\left[\ln\left\lvert\frac{C(x+1)(x-1)}{(2x+3)^2}\right\rvert\right]$ or $\left[\ln\left\lvert\frac{\sqrt{C(x+1)(x-1)}}{(2x+3)}\right\rvert\right]$	A1	cao accept $+C$
			A0 if no C. ISW

Q	Solution	Mark	Notes
6(a)	$T_{12} = 10 + (12 - 1) \times 0.2$	M1	use of $a + (n - 1)d$ Allow $d = 20$ for M1. Implied by correct answer.
	$T_{12} =$ £12.20	A1	
6(b)	$(954 =) \frac{n}{2}[2 \times 10 + (n - 1) \times 0.2]$	M1	use of $\frac{n}{2}[2a + (n - 1)d]$ Allow $d = 20$ for M1.
	$9540 = n[100 + n - 1]$		
	$n^2 + 99n - 9540 = 0$	m1	equating to 954 and writing as quadratic Implied by $n = 60$
	$(n - 60)(n + 159) = 0$		
	$n = 60$	A1	cao Dependent on M1 A0 if $n = -159$ present in final answer
	60 (months)		

Q	Solution	Mark	Notes
7	$x^2 = 8\sqrt{x}$ or $y = \left(\frac{y^2}{64}\right)^2$	M1	equating y's
	$x^4 = 64x$ or $y^4 = 4096y$		
	$x(x^3 - 64) = 0$ or $y(y^3 - 4096) = 0$	A1	oe e.g. $x^{\frac{3}{2}} = 8$
	$x = (0,)\ 4$ or $y = (0,)\ 16$	A1	si, 0 not required.
	Area = $\int_0^4 \left(8x^{\frac{1}{2}} - x^2\right) \mathrm{d}x$	M1	oe allow $x^2 - 8x^{\frac{1}{2}}$
			limits not required
	Area = $\left[\frac{16}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right]_0^4$	A1	one correct term
			Must be seen
		A1	other correct term
	Area = $\frac{16}{3} \times 8 - \frac{1}{3} \times 64$		
	Area = $\frac{64}{3}$	A1	cso
			A0 if integral gives negative answer, unless corrected without any incorrect statements.

Alternative Solution for last 4 marks

Solution	Mark	Notes
$A = \int_0^4 8x^{\frac{1}{2}}\, \mathrm{d}x$	(M1)	
$= \left[\frac{16}{3}x^{\frac{3}{2}}\right]_0^4$	(A1)	Must be seen
$= \frac{16}{3} \times 8$ $(= \frac{128}{3})$		
$B = \int_0^4 x^2\, \mathrm{d}x$		Award M1 if not awarded previously
$= \left[\frac{1}{3}x^3\right]_0^4$	(A1)	Must be seen
$(= \frac{64}{3})$		
Required area = $A - B = \frac{64}{3}$	(A1)	

Note: Answer only, M1 A0 A0 A0

Q	Solution	Mark	Notes
8	$\frac{2-x}{\sqrt{1+3x}} = (2-x)(1+3x)^{-1/2}$		
	$(1+3x)^{-1/2} = (1 + \left(-\frac{1}{2}\right)(3x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(3x)^2 + \ldots)$	B1	$1 + \left(-\frac{1}{2}\right)(3x)$
		B1	$\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(3x)^2$
	$\frac{2-x}{\sqrt{1+3x}} = (2-x)(1 - \frac{3}{2}x + \frac{27}{8}x^2 + \ldots.)$		
	$= 2 - 3x + \frac{27}{4}x^2 - x + \frac{3}{2}x^2 + \ldots$		
	$= 2 - 4x + \frac{33}{4}x^2 + \ldots$	B3	B1 each term
			Ignore further terms, ISW
	Expansion valid for $\|3x\| < 1$		
	$\|x\| < \frac{1}{3}$ or $-\frac{1}{3} < x < \frac{1}{3}$	B1	B1 for $x < \frac{1}{3}$ and $x > -\frac{1}{3}$
			B0 anything else
	When $x = \frac{1}{22}$,		
	$\frac{2-\frac{1}{22}}{\sqrt{1+\frac{3}{22}}} \approx 2 - \frac{4}{22} + \frac{33}{4}\left(\frac{1}{22}\right)^2$	M1	sub into LHS and RHS
	$\frac{\frac{43}{22}}{\frac{5\sqrt{22}}{22}} = \frac{43}{5\sqrt{22}} \approx \frac{323}{176}$ or $\frac{43\sqrt{22}}{110} \approx \frac{323}{176}$		
	$\sqrt{22} \approx \frac{7568}{1615}$ or $\frac{1615}{344}$	A1	cao

(= 4.686068111…, or 4.694767442…, actual value is 4.69041576…)

Special case for $(1 + 3x)^{1/2}$ used

$(1+3x)^{1/2} = (1 + \left(\frac{1}{2}\right)(3x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(3x)^2 + \ldots)$	(B0)	
	(B0)	
$\frac{2-x}{\sqrt{1+3x}} = (2-x)(1 + \frac{3}{2}x - \frac{9}{8}x^2 + \ldots.)$		
$= 2 + 3x - \frac{9}{4}x^2 - x - \frac{3}{2}x^2 + \ldots$		
$= 2 + 2x - \frac{15}{4}x^2 + \ldots$	(B3)	B1 each term
		Ignore further terms, ISW
Expansion valid for $\lvert 3x \rvert < 1$		
$\lvert x \rvert < \frac{1}{3}$ or $-\frac{1}{3} < x < \frac{1}{3}$	(B1)	B1 for $x < \frac{1}{3}$ and $x > -\frac{1}{3}$
		B0 anything else
Correct substitution	(M1)	
	(A0)	

Q	Solution	Mark	Notes
9(a)	$u_1 = \sin\left(\frac{\pi}{2}\right) = 1$		
	$u_2 = \sin\left(\frac{2\pi}{2}\right) = 0$		
	$u_3 = \sin\left(\frac{3\pi}{2}\right) = -1$		
	$u_4 = \sin\left(\frac{4\pi}{2}\right) = 0$		
	$u_5 = \sin\left(\frac{5\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$	B1	All 5 terms
	Sequence is periodic (with period 4)	B1	Condone 'Repeats every 4 terms' or 'Oscillates'
9(b)	$u_5 = 17$	B1	
	$(u_5 = 17)$, $u_4 = 9$, $u_3 = 5$, $u_2 = 3$, $u_1 = 2$	B1	
	Sequence is increasing.	B1	Accept 'Divergent'

Q	Solution	Mark	Notes
10	$\frac{6x^5-17x^4-5x^3+6x^2}{(3x+2)} = \frac{(x^2)(6x^3-17x^2-5x+6)}{(3x+2)}$	M1	or removing x^2 from pentic
	$= \frac{(x^2)(3x+2)(2x^2-7x+3)}{(3x+2)}$	M1	divide by $(3x + 2)$, or realising $(3x + 2)$ is a factor of the cubic **and** cancelling
		A1	Sight of $(2x^2 - 7x + 3)$
	$= x^2(2x - 1)(x - 3) = 0$	A1	Must be seen
	$x = 0$(twice), $\frac{1}{2}$, 3.	A1	cao A0 if $-\frac{2}{3}$ present

Note: $(6x^3 - 17x^2 - 5x + 6) = (x - 3)(6x^2 + x - 2)$

$(6x^3 - 17x^2 - 5x + 6) = (2x - 1)(3x^2 - 7x - 6)$

Alternative Solution

Q	Solution	Mark	Notes
10	$\frac{6x^5-17x^4-5x^3+6x^2}{(3x+2)} = \frac{(x^2)(6x^3-17x^2-5x+6)}{(3x+2)}$	(M1)	or removing x^2 from pentic
	$= \frac{(x^2)(3x+2)(2x^2-7x+3)}{(3x+2)}$	(M1)	any linear factor or divide by $(3x + 2)$
		(A1)	Sight of $(2x^2 - 7x + 3)$ oe or second factor from factor theorem
	$= x^2(2x - 1)(x - 3) = 0$	(A1)	$(3x + 2)$ must be cancelled or solution discarded
	$x = 0$ (twice), $\frac{1}{2}$, 3.	(A1)	cao A0 if $-\frac{2}{3}$ present

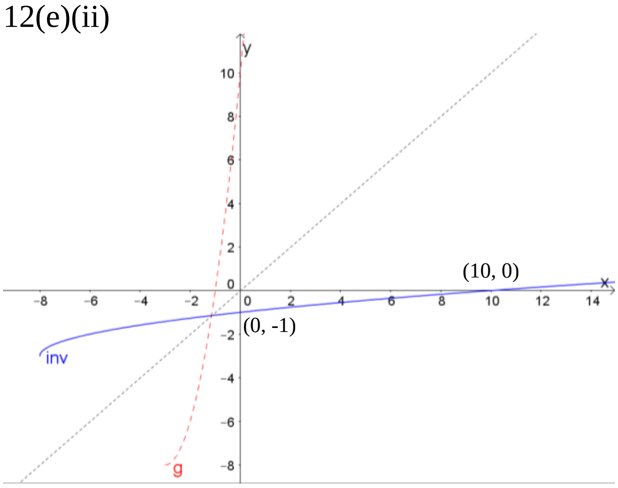
Note: $(6x^3 - 17x^2 - 5x + 6) = (x - 3)(6x^2 + x - 2)$

$(6x^3 - 17x^2 - 5x + 6) = (2x - 1)(3x^2 - 7x - 6)$

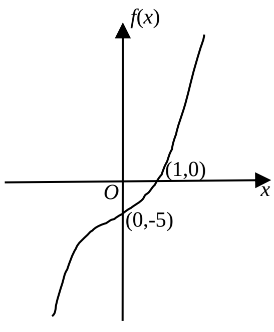
Q	Solution	Mark	Notes
11(a)	$9\cos x + 40\sin x = R\cos x\cos\alpha + R\sin x\sin\alpha$		
	$R\cos\alpha = 9$ and $R\sin\alpha = 40$	M1	implied by correct α if nothing seen. M0 for incorrect equations
	$R = \sqrt{9^2 + 40^2} = 41$	B1	
	$\alpha = \tan^{-1}\left(\frac{40}{9}\right) = 77.32°$	A1	accept 1.349 rad, not 1.349 ft R if $\alpha = \sin^{-1}\left(\frac{40}{R}\right) = \cos^{-1}\left(\frac{9}{R}\right)$
	$9\cos x + 40\sin x \equiv 41\cos(x - 77.32°)$		
11(b)	$y = \frac{12}{9\cos x + 40\sin x + 47}$		
	Maximum y when denominator is minimum,		
	i.e. when $\cos(x – 77.32°) = –1$	M1	implied by correct max
	Max $y \left(= \frac{12}{-41+47}\right) = 2$	A1	ft R

Q	Solution	Mark	Notes
12(a)	$ff(p) = f(0) = 10$	B1	
12(b)	$2x^2 + 12x + 10 = 0$ $2(x^2 + 6x + 5) = 0$ $2(x + 5)(x + 1) = 0$	M1	may be implied by solution
	$p = -5,\ q = -1$	A1	both
12(c)	$f(x) = 2[x^2 + 6x + 5]$		
	$= 2[(x + 3)^2 - 4]$	M1	condone absence of '2'
	$= 2(x + 3)^2 - 8$	A1	cao
	Min point at $(-3, -8)$	B1	
12(d)	$f(x)$ is not a one-to-one function (on its domain).	B1	
12(e)(i)	Let $y = 2(x + 3)^2 - 8$		
	$(x + 3)^2 = \frac{y+8}{2}$	M1	ft similar form from (c)
	$x = -3 \pm \sqrt{\frac{y+8}{2}}$	A1	Condone $x = -3 + \sqrt{\frac{y+8}{2}}$
	since $x \geq -3$, $x = -3 + \sqrt{\frac{y+8}{2}}$	A1	Must discard negative root
	$g^{-1}(x) = -3 + \sqrt{\frac{x+8}{2}}$	A1	interchange x and y, could be done earlier

12(e)(ii)



Mark	Notes
B1	Correct shape
B1	(10, 0) (0, -1), cao

Q	Solution	Mark	Notes
13(a)	$f'(x) = 6x^2 + 3$	B1	
	Hence $f'(x) > 0$ for all x,		
	i.e. $f(x)$ does not have a stationary point.	E1	oe
			e.g. $f'(x) = 0$ has no real roots
			discriminant = $0^2 – 4(6)(3) < 0$, no real roots
13(b)	$f''(x) = 12x$	M1	
	At point of inflection $f''(x) = 0$, $x = 0$	m1	
	$f'(x) > 0$ when $x < 0$ and when $x > 0$.		
	Therefore, when $x = 0$,		
	there is a point of inflection.	A1	oe cubic curve no max/min
			must have a point of inflection.
			OR $x > 0$, $f''(x) > 0$; $x <0$, $f''(x) < 0$
	The point of inflection is (0, –5)	B1	
13(c)			
		G1	cubic curve no max/min
			ft point in (b) coords not required.
			(1,0) not required.

Q Solution Mark Notes

14	$I = [\pm \cos x \,.\, x^2]_0^\pi - \int_0^\pi \pm \cos x \,.\, 2x \,\mathrm{d}x$	M1	attempt at parts, 2 terms, at least one term correct. Limits not required
	$I = [-\cos x \,.\, x^2]_0^\pi - \int_0^\pi -\cos x \,.\, 2x \,\mathrm{d}x$	A1	
	$I = [-\cos x \,.\, x^2]_0^\pi + [\sin x.\, 2x]_0^\pi$		
	$- \int_0^\pi 2\sin x \,\mathrm{d}x$	A1	correct integration of $\int_0^\pi \pm \cos x \,.\, 2x \,\mathrm{d}x$
	$I = [-\cos x \,.\, x^2]_0^\pi + [2\sin x.\, x]_0^\pi + [2\cos x]_0^\pi$	A1	correct integration of $\int_0^\pi \pm \sin x \,\mathrm{d}x$
	$I = [2x\sin x + (2 - x^2)\cos x]_0^\pi$		
	$I = \pi^2 + 0 + 2(-1 - 1)$	m1	correct use of correct limits Implied by correct answer
	$I = \pi^2 - 4$ (= 5.87)	A1	cao

<u>Note</u>

No marks for answer unsupported by workings.

If integration is incorrect and answer of 5.87 seen with **no working**, m0 A0. If substitution seen m1 is available.

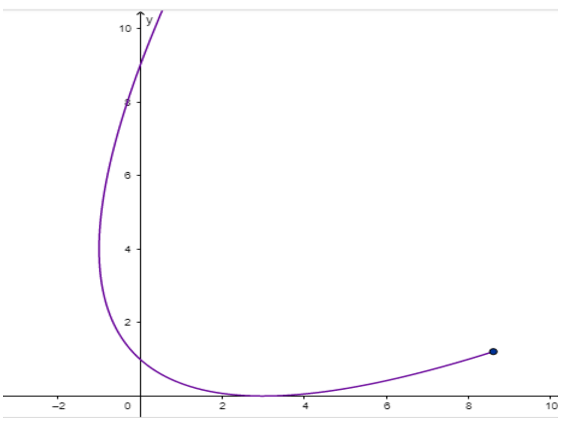
Be careful of use of calculators to obtain correct answer after incorrect integration.

Condone missing d*x*.

M1A0 only for $I = \left[\sin x \,.\, \frac{x^3}{3}\right]_0^\pi - \int_0^\pi \frac{x^3}{3} \cos x \,\mathrm{d}x$

Q	Solution	Mark	Notes
15(a)	$y = \sqrt{16 - x^2}$ OR $A = 2xy$	B1	
	$A = 2x\sqrt{16 - x^2}$	B1	
15(b)	$\frac{dA}{dx} = \frac{d}{dx}[2x(16 - x^2)^{1/2}]$	M1	$f(x)\ (16 - x^2)^{1/2} + 2xg(x)$
			M0 if $f(x) = 0$ or 1 or $g(x) = 0$ or 1
			Only ft if product with $Bx\sqrt{K - x^2}$
	$\frac{dA}{dx} = 2(16 - x^2)^{1/2} + 2x \times \frac{1}{2}(16 - x^2)^{-1/2}(-2x)$	A1A1	one each term, ft (a)
	$\frac{dA}{dx} = \frac{4}{(16-x^2)^{1/2}}[8 - x^2]$		
	At max, $\frac{dA}{dx} = 0$	m1	
	$x^2 = 8$	A1	cao
	$x = 2\sqrt{2}$ (-ve value inadmissible)		
	$y = \sqrt{16 - x^2} = \sqrt{16 - 8} = 2\sqrt{2}$	A1	cao accept $y^2 = 8$
	therefore $y = x$.		
	Justification of maximum	B1	$\frac{d^2A}{dx^2} = -22$ when $x = 2\sqrt{2}$
	OR		
	$A^2 = 4x^2(16 - x^2) = 64x^2 - 4x^4$		
	$\frac{dA^2}{dx} = 128x - 16x^3$	(M1A1A1)	
	At max, $\frac{dA^2}{dx} = 0$	(m1)	
	$x^2 = 8$, $x = 2\sqrt{2}$ (-ve value inadmissible)	(A1)	cao
	$y = \sqrt{16 - x^2} = \sqrt{16 - 8} = 2\sqrt{2}$	(A1)	cao accept $y^2 = 8$
	therefore $y = x$.		
	Justification of maximum	(B1)	$\frac{d^2A^2}{dx^2} = -256$ when $x = 2\sqrt{2}$

Q	Solution	Mark	Notes
16(a)	Where C meets the y-axis,		
	$3 - 4t + t^2 = 0$	M1	
	$(t - 1)(t - 3) = 0$		
	$t = 1$, point is (0, 9)	A1	or $t = 1, 3$
	$t = 3$, point is (0, 1)	A1	all correct
16(b)	$\frac{dy}{dt} = -2(4 - t)$	B1	
	$\frac{dx}{dt} = -4 + 2t$	B1	
	$\frac{dy}{dx} = \frac{-2(4-t)}{-4+2t}$	B1	ft their dy/dt and dx/dt
	Note: May be seen in (a)		
	At stationary point, $\frac{-2(4-t)}{-4+2t} = 0$	M1	
	$t = 4$		
	At stationary point, $y = (4 - 4)^2 = 0$.		
	Hence the x-axis is a tangent to the curve C.	A1	



Q	Solution	Mark	Notes
17(a)	$\cos(\alpha - \beta) + \sin(\alpha + \beta)$		
	$= \cos\alpha\cos\beta + \sin\alpha\sin\beta + \sin\alpha\cos\beta + \cos\alpha\sin\beta$	B1	expand $\cos(\alpha - \beta)$, $\sin(\alpha + \beta)$
	$=\cos\alpha(\cos\beta + \sin\beta) + \sin\alpha(\cos\beta + \sin\beta)$		
	$=(\cos\alpha + \sin\alpha)(\cos\beta + \sin\beta)$	B1	convincing
	OR		
	$(\cos\alpha + \sin\alpha)(\cos\beta + \sin\beta)$		
	$= \cos\alpha\cos\beta + \cos\alpha\sin\beta + \sin\alpha\cos\beta + \sin\alpha\sin\beta$	(B1)	remove brackets
	$= \cos\alpha\cos\beta + \sin\alpha\sin\beta + \sin\alpha\cos\beta + \cos\alpha\sin\beta$		
	$= \cos(\alpha - \beta) + \sin(\alpha + \beta)$	(B1)	convincing
	OR		
	$\cos(\alpha - \beta) + \sin(\alpha + \beta)$		
	$= \cos\alpha\cos\beta + \sin\alpha\sin\beta + \sin\alpha\cos\beta + \cos\alpha\sin\beta$	(B1)	expand $\cos(\alpha - \beta)$, $\sin(\alpha + \beta)$
	$(\cos\alpha + \sin\alpha)(\cos\beta + \sin\beta)$		
	$= \cos\alpha\cos\beta + \cos\alpha\sin\beta + \sin\alpha\cos\beta + \sin\alpha\sin\beta$	(B1)	remove brackets
	Hence $\cos(\alpha - \beta) + \sin(\alpha + \beta)$		
	$= (\cos\alpha + \sin\alpha)(\cos\beta + \sin\beta)$		

Q	Solution	Mark	Notes
17(b)(i)	Put $\alpha = 4\theta$, $\beta = \theta$	M1	
	$\cos(4\theta - \theta) + \sin(4\theta + \theta)$		
	$= (\cos4\theta + \sin4\theta)(\cos\theta + \sin\theta)$		
	$\frac{\cos3\theta+\sin5\theta}{\cos4\theta+\sin4\theta} = \cos\theta + \sin\theta$	A1	convincing
17(b)(ii)	When $\theta = \frac{3\pi}{16}$,		
	$\cos4\theta + \sin4\theta = \cos\frac{3\pi}{4} + \sin\frac{3\pi}{4} = 0$		
	So $\frac{\cos3\theta+\sin5\theta}{\cos4\theta+\sin4\theta}$ is undefined.	B1	oe
	OR		
	$\cos4\theta + \sin4\theta \neq 0$		
	$\tan4\theta \neq -1$		
	$4\theta \neq \frac{3\pi}{4}$		
	$\theta \neq \frac{3\pi}{16}$	B1	

Q	Solution	Mark	Notes
18(a)	Put $u = x + 3$	B1	
	$\int \frac{x^2}{(x+3)^4}\,\mathrm{d}x = \int \frac{(u-3)^2}{u^4}\,\mathrm{d}u$	M1	Allow one slip
	$= \int \frac{u^2-6u+9}{u^4}\,\mathrm{d}u$		
	$= \int (u^{-2} - 6u^{-3} + 9u^{-4})\,\mathrm{d}u$	A1	integrable form ft $(u + 3)$ only
	$= \frac{u^{-1}}{-1} - \frac{6u^{-2}}{-2} + \frac{9u^{-3}}{-3}\,(+C)$	A1	correct integration ft $(u + 3)$ only
	$= -\frac{1}{u} + \frac{3}{u^2} - \frac{3}{u^3}\,(+C)$		
	$= -\frac{1}{x+3} + \frac{3}{(x+3)^2} - \frac{3}{(x+3)^3} + C$	A1	cao Correct expression in terms of x Must include + C
18(b)	$\int_0^1 \frac{x^2}{(x+3)^4}\,\mathrm{d}x = \left[-\frac{1}{x+3} + \frac{3}{(x+3)^2} - \frac{3}{(x+3)^3}\right]_0^1$		
	$= \left(-\frac{1}{4} + \frac{3}{16} - \frac{3}{64}\right) - \left(-\frac{1}{3} + \frac{1}{3} - \frac{1}{9}\right)$	M1	correct use of correct limits ft for equivalent difficulty for M1 only
	$= \frac{1}{576}$ (= 0.001736)	A1	cao No workings, 0 marks
OR			
	$\int_0^1 \frac{x^2}{(x+3)^4}\,\mathrm{d}x = \left[-\frac{1}{u} + \frac{3}{u^2} - \frac{3}{u^3}\right]_3^4$		
	$= \left(-\frac{1}{4} + \frac{3}{16} - \frac{3}{64}\right) - \left(-\frac{1}{3} + \frac{1}{3} - \frac{1}{9}\right)$	(M1)	correct use of correct limits ft for equivalent difficulty for M1 only
	$= \frac{1}{576}$ (= 0.001736)	(A1)	cao No workings, 0 marks