GCE A LEVEL

1305U60-1

Z22-1305U60-1

THURSDAY, 23 JUNE 2022 – AFTERNOON

FURTHER MATHEMATICS – A2 unit 6
FURTHER MECHANICS B

1 hour 45 minutes

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Take g as $9{\cdot}8\,\text{ms}^{-2}$.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Answers without working may not gain full credit.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 80.
The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. A particle is moving along the x-axis. At time t seconds the particle is x metres from the origin, *O*, and its velocity $v\,\text{ms}^{-1}$ is given by

$$v = \frac{24}{4x+9}.$$

(a) Find, in terms of x, an expression for the acceleration of the particle at time t s. [3]

(b) At $t = T$ the acceleration of the particle is $-\frac{4}{3}\,\text{ms}^{-2}$.

(i) Determine the value of x when $t = T$.

(ii) Given that $x = -2$ when $t = 0$, find an expression for t in terms of x and hence find the value of T. [9]

2. A particle *P* moves along the x-axis such that its position x metres, after t seconds, is given by

$$x = \sin(\pi t) + \sqrt{3}\cos(\pi t).$$

(a) (i) Show that the motion of the particle *P* is Simple Harmonic. State the value of x at the centre of motion.

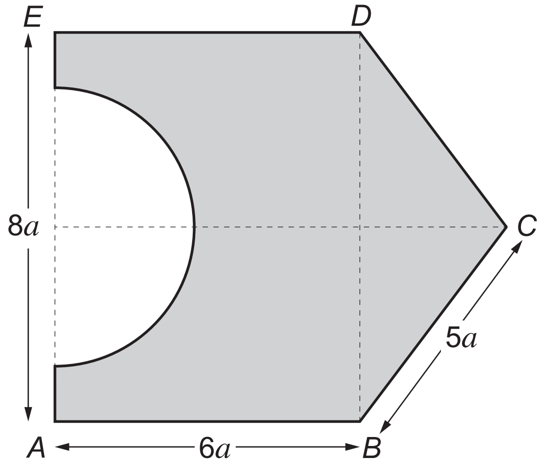
(ii) Show that the period of the motion of *P* is 2 s and determine the amplitude. [8]

Suppose that another particle *Q* is introduced so that it also moves along the x-axis with Simple Harmonic Motion with centre of motion, *O*, and period equal to that of particle *P*. When $t = 0$, the particle *Q* is at *O* and when it is $2\sqrt{3}$ m from *O* its speed is $2\pi\,\text{ms}^{-1}$.

(b) Find the amplitude of particle *Q*. [3]

(c) Determine the time when particles *P* and *Q* first meet. [4]

3. The diagram below shows a lamina *ABCDE* which is made of a uniform material. It consists of a rectangle *ABDE* with $AB = 6a$ and $AE = 8a$, together with an isosceles triangle *BCD* with $BC = DC = 5a$. A semicircle, with its centre at the midpoint of *AE* and radius $3a$, is removed from *ABDE*.



(a) Write down the distance of the centre of mass of the lamina *ABCDE* from *AB*. [1]

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(b) Show that the distance of the centre of mass of the lamina *ABCDE* from *AE* is $\frac{140}{40-3\pi}a$. [7]

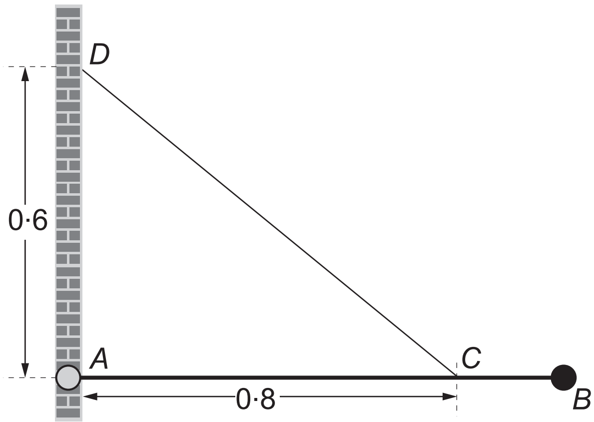
(c) The lamina *ABCDE* is freely suspended from the point *D* and hangs in equilibrium.

(i) Calculate the angle that *BD* makes with the vertical.

(ii) The mass of the lamina is M. When a particle of mass kM is attached at the point *C*, the lamina hangs in equilibrium with *AB* horizontal. Determine the value of k. [6]

TURN OVER

4. The diagram below shows a uniform rod *AB*, of weight 10 N, hinged to a vertical wall at *A*. The rod is held in a horizontal position by means of a light inextensible string. One end of the string is attached to a point *C* on the rod and the other end is attached to a point *D* on the wall. The point *D* is 0·6 m vertically above *A* and the length of *AC* is 0·8 m. A particle of weight 25 N is attached to the rod at *B* and the tension in the string is 75 N.



(a) Find the length of the rod *AB*. [4]

(b) Calculate the magnitude and direction of the reaction at the hinge at *A*. [8]

5. Two smooth spheres *A* and *B*, of equal radii, are moving on a smooth horizontal plane when they collide. Immediately after the collision sphere *A* has velocity $(-2\mathbf{i} - 5\mathbf{j})\,\text{ms}^{-1}$ and sphere *B* has velocity $(\mathbf{i} + 3\mathbf{j})\,\text{ms}^{-1}$. When the spheres collide, their line of centres is parallel to the vector $\mathbf{i}$ and the coefficient of restitution between the spheres is $\frac{2}{5}$. Sphere *A* has mass 4 kg and sphere *B* has mass 2 kg.

(a) Find the velocity of *A* and the velocity of *B* immediately before the collision. [7]

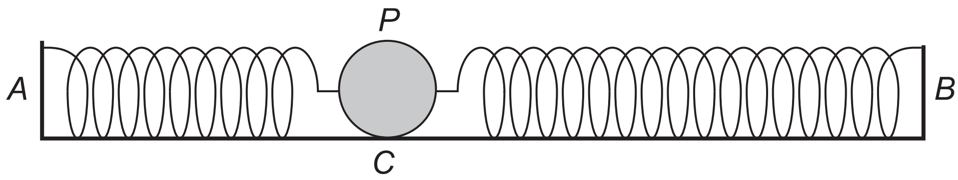
After the collision, sphere *A* continues to move with velocity $(-2\mathbf{i} - 5\mathbf{j})\,\text{ms}^{-1}$ until it collides with a smooth vertical wall. The impulse exerted by the wall on *A* is $32\mathbf{j}$ Ns.

(b) State whether the wall is parallel to the vector $\mathbf{i}$ or to the vector $\mathbf{j}$. Give a reason for your answer. [1]

(c) Find the speed of *A* after the collision with the wall. [3]

(d) Calculate the loss of kinetic energy caused by the collision of sphere *A* with the wall. [2]

6. The diagram shows a particle P, of mass 4 kg, lying on a smooth horizontal surface. It is attached by two light springs to fixed points A and B, where $AB = 2{\cdot}8$ m. Spring AP has natural length $0{\cdot}8$ m and modulus of elasticity 60 N. Spring PB has natural length $1{\cdot}2$ m and modulus of elasticity 30 N.



When P is in equilibrium, it is at the point C.

(a) Show that $AC = 1$ m. [4]

(b) The particle P is pulled horizontally and is initially held at rest at the midpoint of AB. The system is then released.

(i) Show that P performs Simple Harmonic Motion about centre C and find the period of its motion.

(ii) Determine the shortest time taken for P to reach a position where there is no tension in the spring AP. [10]

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