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Z22-C300UA0-1

## FRIDAY, 20 MAY 2022 - MORNING

## MATHEMATICS - Component 1

Non-Calculator Mathematics HIGHER TIER

2 hours 15 minutes

## ADDITIONAL MATERIALS

An additional formulae sheet.
The use of a calculator is not permitted in this examination. A ruler, protractor and a pair of compasses may be required.

## INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Do not use gel pen or correction fluid.
You may use a pencil for graphs and diagrams only.
Write your name, centre number and candidate number in the spaces at the top of this page.
Answer all the questions in the spaces provided.
If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

## INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.
Unless stated, diagrams are not drawn to scale.
Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the need for good English and orderly, clear presentation in your answers.


| For Examiner's use only |  |  |
| :---: | :---: | :---: |
| Question | Maximum Mark | Mark Awarded |
| 1. | 3 |  |
| 2. | 2 |  |
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| 20. | 4 |  |
| 21. | 8 |  |
| 22. | 4 |  |
| 23. | 8 |  |
| 24. | 3 |  |
| 25. | 3 |  |
| Total | 120 |  |

## Formula list

## Area and volume formulae

Where $r$ is the radius of the sphere or cone, $l$ is the slant height of a cone and $h$ is the perpendicular height of a cone:

$$
\begin{aligned}
& \text { Curved surface area of a cone }=\pi r l \\
& \text { Surface area of a sphere }=4 \pi r^{2} \\
& \text { Volume of a sphere }=\frac{4}{3} \pi r^{3} \\
& \text { Volume of a cone }=\frac{1}{3} \pi r^{2} h
\end{aligned}
$$

## Kinematics formulae

Where $a$ is constant acceleration, $u$ is initial velocity, $v$ is final velocity, $s$ is displacement from the position when $t=0$ and $t$ is time taken:

$$
\begin{gathered}
v=u+a t \\
s=u t+\frac{1}{2} a t^{2} \\
v^{2}=u^{2}+2 a s
\end{gathered}
$$

1. In 2019,

- $€ 1=£ 0.90$,
- $\$ 1.25=£ 1$.

In 2019, a silver pencil cost $€ 110$ in Germany. The same pencil cost $\$ 125$ in the USA.

In which country was the pencil cheaper?


You must show all your working.
$\qquad$
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2. The diagram shows a parallelogram, $A B C D$ and the diagonal $A C$.


Diagram not drawn to scale

Tick ( $\mathcal{\prime}$ ) the two correct statements.

| $A \widehat{B C}$ is not equal to $C \widehat{D A}$ |  |
| :--- | :--- |
| $A B=D C$ and $A D=B C$ and $A C$ is a side of both triangle $A B C$ and triangle $C D A$ |  |
| Triangle $A B C$ is similar to triangle $C D A$ with enlargement scale factor 0.5 |  |
| Triangle $A B C$ is not congruent to triangle $C D A$ |  |
| Triangle $A B C$ is congruent to triangle $C D A$ |  |
| $A B$ represents the shortest distance from $B$ to $A C$ |  |

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3. The diagram shows a cone placed with its circular base on a table. It has

- base radius 15 cm ,
- height 30 cm .

(a) Work out the volume of this cone. Give your answer as a multiple of $\pi$.

Diagram not drawn to scale
(b) On the 1 cm grid opposite, make an accurate scale drawing of the plan and side elevation of this cone.

Use the ratio
actual cone : scale drawing $=5: 1$.

## Side elevation

4. A tank contains 225 litres of water.

A tap at the bottom is opened so that water flows out at a constant rate of 5 litres every minute until the tank is empty.
(a) On the graph paper below, draw a line to show the volume of water in the tank at any time after the tap has been opened.

(b) How many minutes does it take for the volume of water in the tank to decrease by $40 \%$ of the original volume?
$\qquad$
$\qquad$
5. The probability that Kathy cycles to work on Monday is $0 \cdot 6$.

If she cycles to work on Monday, the probability that she cycles to work on Tuesday is 0.3. If she does not cycle to work on Monday, the probability that she does not cycle to work on Tuesday is $0 \cdot 1$.
(a) Complete the tree diagram.

Monday
Tuesday

(b) Calculate the probability that Kathy cycles to work on both Monday and Tuesday.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(c) Calculate the probability that Kathy does not cycle to work on either day.
6. In a factory, 6 identical machines can make 3000 erasers in 2 hours.

How long would it take 8 of these machines to make 1000 erasers?
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7. (a) Expand and simplify $(4 x+5)(2 x-1)$.
$\qquad$
$\qquad$
$\qquad$
(b) (i) Factorise $x^{2}-10 x+21$.
$\qquad$
$\qquad$
$\qquad$
(ii) Use your answer to part (b)(i) to write down the solutions of the equation $x^{2}-10 x+21=0$.

$$
x=\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots
$$

8. Vikram wanted to find out how many moths there were in a small woodland.

One night, Vikram captured a random sample of 12 moths and marked them.
He then released them back into the woodland.
The next night, Vikram captured a second random sample of 30 moths. He found that 9 of the moths in the second sample had been marked.

Vikram estimated that there were 40 moths in the woodland.
(a) Show that Vikram's estimate of the number of moths was correct.
$\qquad$
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$\qquad$
(b) Comment on how reliable Vikram's estimate was likely to be.
$\qquad$
$\qquad$
$\qquad$
9. Deena sells a painting for $£ 8690$.

This is $10 \%$ more than she originally paid for it.
How much did Deena pay for the painting?
$\qquad$
$\qquad$
How much did Deena pay for the painting?
10. (a) Write 378 as a product of its prime factors. Give your answer in index form.

Product of prime factors in index form
(b) Use prime factors to prove that 1 is the only common factor of 378 and 275.
11. (a) The point $O$ is the origin.

The points $O, P$, and $R$ are such that $\mathbf{O P}=\binom{1}{3}$ and $\mathbf{O R}=\binom{5}{-1}$.
(i) Find PR.

You may use this grid to help you.


$$
\mathbf{P R}=(
$$

(ii) The point $Q$ is such that, when taken in a clockwise direction, the points $O, P, Q$ and $R$ form a parallelogram.

Find $\mathbf{O Q}$.
You may use the grid above to help you.

$$
\mathbf{O Q}=(
$$

(b) The grid shows the vectors $\mathbf{a}$ and $\mathbf{b}$.


Mitch wants to draw the vector $2 \mathbf{a}+\mathbf{b}$.
His diagram is shown below.


Make two criticisms of Mitch's diagram.
Criticism 1 $\qquad$
$\qquad$
$\qquad$
Criticism 2 $\qquad$
$\qquad$
$\qquad$
12. (a) Three quantities $A, B$ and $C$ are in the ratio

$$
A: B: C \text { is } 13: 7: 2
$$

Given that $A-B=48$, find the value of $A+B+C$.
$\qquad$
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(b) $\quad x$ cars travel a total of 1000 kilometres.

Each car uses $f$ litres of fuel.
Each car travels the same number of kilometres per litre.
Find an algebraic expression for the number of kilometres per litre travelled by each car.
13. The table shows the population and area of land for country $X$.

|  | Population | Area $\left(\mathrm{km}^{2}\right)$ |
| :---: | :---: | :---: |
| Country X | $2.16 \times 10^{7}$ | 3000 |

Population density can be measured in number of people per square kilometre.
The population density of country Y is 8000 people per $\mathrm{km}^{2}$.
Which country has the greater population density and by how much is it greater?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
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$\qquad$

The population density of country
is greater by $\qquad$ people per $\mathrm{km}^{2}$.
14. Rearrange this formula to make $a$ the subject.

$$
\frac{a^{3} b}{7}+5=c
$$

15. 



Diagram not drawn to scale
$P, Q, R$ and $S$ are points on a circle with centre $O$.
The line $T U$ is a tangent to the circle at the point $S$.
$P \widehat{O} S=130^{\circ}$ and $P \widehat{Q} R=120^{\circ}$.
Show that $x=55^{\circ}$.
Give a reason for each step of your answer.
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16. Brian and Yvonne are gardeners. They each have an orchard of fully-grown apple trees.
(a) The table shows information about the height, in cm , of Yvonne's 43 apple trees on 1st September.

| Tree height, <br> $h(\mathrm{~cm})$ | $200<h \leqslant 250$ | $250<h \leqslant 260$ | $260<h \leqslant 290$ | $290<h \leqslant 300$ | $300<h \leqslant 320$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Frequency | 5 | 8 | 12 | 13 | 5 |
| Frequency <br> density |  |  |  |  |  |

(i) Complete the frequency density row in the table above.
$\qquad$
$\qquad$
$\qquad$
(ii) Draw a histogram to illustrate the data in the table.

Frequency
density

17. The diagram shows an equilateral triangle with side $2 x \mathrm{~cm}$.


## Diagram not drawn to scale

The height of the triangle is $h \mathrm{~cm}$.
Find and simplify an expression for $h$ in terms of $x$.
$\qquad$
$\qquad$
18. (a) Write $5^{3} \div 5^{-4}$ as a single power of 5 .
(b) Calculate the value of $10000^{\frac{3}{4}}$.
(c) Simplify $\sqrt{49 \times 10^{2 n}}$.
19. (a) Write $\frac{1}{27}$ as a recurring decimal.
(b) By writing $1 \cdot 2 \dot{4} \dot{3}$ as a fraction, calculate $1 \cdot 2 \dot{4} \dot{3}-\frac{8}{9}$.

Give your answer as a fraction.
$\qquad$
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20. $g(x)=\frac{x}{2}$

$$
h(x)=x^{3}
$$

(a) Find $h g\left(\frac{1}{3}\right)$.
(b) Solve $h^{-1}(x)=-2$.
21. (a) (i) On the axes below, sketch the graph of $y=\cos x$ for $-180^{\circ} \leqslant x \leqslant 180^{\circ}$.

(ii) Solve $\cos x=\frac{\sqrt{3}}{2}$ for $-180^{\circ} \leqslant x \leqslant 180^{\circ}$.
(b) The diagram shows a sketch of the graph of $y=f(x)$. The point $A$ has coordinates $(-2,0)$ and the point $B$ has coordinates $(0,3)$.

$\qquad$
$\qquad$
$\qquad$


Examiner


You must indicate the coordinates of the new positions of the points $A$ and $B$.

22. A game of chance at a school fete is played with the following rules.

- There are five identical balls numbered from 1 to 5 in a bag.
- A player takes two balls from the bag at random.
- A player wins a prize when their two balls are numbered 2 and 4.
- At the end of each game, both balls are put back in the bag.
(a) Olivia plays the game once.

What is the probability that she wins a prize?
(b) Alex plays the game and stops playing when he wins.

What is the probability that he only plays the game twice?
23. (a) Write $7 \sqrt{3}(5 \sqrt{3}-4)+\sqrt{27}$ in the form $a+b \sqrt{3}$, where $a$ and $b$ are integers.
(b) In this question all lengths are in centimetres.


## Diagram not drawn to scale

This shape is made from two rectangles.
The area of this shape is $33 \sqrt{2}-18$.
Showing all your working, find the value of $x$.
Give your answer in the form $c+d \sqrt{2}$, where $c$ and $d$ are integers.
24. A circle has equation $x^{2}+y^{2}=400$.
(a) Write down the length of the radius.
(b) The points $A(12,-16)$ and $B$ lie on the circle.
$A B$ is a diameter of the circle.
(b) The points $A(12,-16)$ and $B$ lie on the circle.
$A B$ is a diameter of the circle.

Find the coordinates of $B$.
25. A hummingbird flies at a speed of $v$ metres per second for $t$ seconds after it has finished feeding from a flower.


The diagram shows the speed of the hummingbird for $5 \leqslant t \leqslant 15$.


When $t=5, v=6$.
The average acceleration of the hummingbird for $5 \leqslant t \leqslant 15$ is $0.5 \mathrm{~m} / \mathrm{s}^{2}$.
Calculate the speed of the hummingbird when $t=15$.
You must show all your working.
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| $\begin{array}{\|l\|} \hline \text { Question } \\ \text { number } \\ \hline \end{array}$ | Additional page, if required. <br> Write the question number(s) in the left-hand margin. |
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